Viscosity Coefficients of Rotating Gases --- Accretion Disks vs Planetary Rings

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As to angular momentum transport in an accretion disc there are obvious errors in some famous textbooks adopting a mean free path argument. If we followed exactly their argument, angular momentum would be transported inwards, of which fact contradicts with our common sense. In the present paper we derive viscosity formula in a rotating gas from the first principle. We employ BGK approximation to the Boltzmann equation to get the Krook equation, which is further approximated by the Chapman-Enskog expansion. We obtain viscosity coefficient as well as thermal conductivity, and we find complex effects in rotating gas

The standard model (alpha disk model) of accretion disks considers that the angular momentum of an accretion disk is transported outwards by action of some kind of viscosity. The original standard model takes turbulent viscosity as ``some kind of viscosity''. With the theory of turbulence being incomplete, the molecular viscosity formula is used for this purpose in astrophysics.

Some textbooks contain confusion with respect to the angular momentum transport of an accretion disk. The famous ``Accretion Power in Astrophysics'' (Frank, King, & Raine, 1985) and ``Accretion Processes in Star Formation'' (Hartmann, 1998) employ a generally used mean free path argument of gas molecules. These authors then draw the accepted conclusion that the angular momentum is transported outwards. Their derivation processes contain obvious errors. Correct calculation of their formulas should lead to the conclusion that the angular momentum is transported inwards from the outer parts of the disk. This conclusion is obviously against our common sense and is not correct. Therefore the mean free path argument could not be applied to a rotating fluid.

On the other hand, some other textbooks adopt the viscosity formula for viscous fluids as it is. Let us recall the microscopic derivation process of the viscosity formula. It starts at the Boltzmann equation. A complete equilibrium, expressed by the Maxwell distribution function, isothermal, iso-density and in a uniform motion, where no force is assumed to act. In deriving the viscosity formula, one assumes:

1) The condition of the fluid deviates a little from the above equilibrium, so that the distribution function deviates a little from the Maxwell distribution;

2) Force does not depend on the velocity;

3) If there were rotation, it should be rigid rotation;

None of these assumptions holds for accretion disks or planetary rings. In the present paper we at first show a viscosity formula for static gases. To this end, we start at the Boltzmann equation, and then simplify the equation into the Krook equation, by applying the BGK approximation, which simplifies the collision term. Assuming that the solution deviates only a little from a local Maxwellian distribution, we subject the distribution function to the Chapman-Enskog expansion, to obtain a distribution function. Based on the distribution function thus obtained, we evaluate the viscosity coefficients and thermal conductivity coefficients.

We then expand our theory to rotating gases, where we use a rotating frame. Centrifugal force and Coriolis force appear as a result. Coriolis force, which depends on velocity, requires a special treatment.

The conclusions drawn by this approach are as follows. The viscosity formula itself is the same as that for non-rotating gas, while the magnitude of the viscosity coefficients depend on tau x Omega , where tau is the mean flight time of gas molecules and Omega the angular velocity of rotation. The viscosity coefficients become smaller in proportion to the square of this value. In addition, there appears a queer effect that a shear stress occurs also in a direction perpendicular to the velocity gradient. For gases in Keplerian rotation, the basic distribution function cannot be expressed by the Maxwell distribution, since the basic state is no longer a rigid-body rotation. These problems are best handled by numerical simulation.