Semi-analytic BIEM scheme for the flat 2-D crack dynamics: the question of numerical stability

Taku Tada[1]

[1] ERI Univ. Tokyo

The boundary integral equation method (BIEM) enables numerical modeling of crack dynamics with high efficiency and accuracy under certain conditions, but it is also prone to numerical instabilities depending on the situation. By simulating the time-domain dynamics of a two-dimensional planar crack using the semi-analytic BIEM, I found out that the numerical stability had a complicated dependence on the combination of two grid parameters, namely the ratio of the discrete time step size to the spatial grid size, and the relative height of the collocation point along the time axis. It was also shown that the scheme was largely unstable in mode II but that it could be stabilized by using the epsilon-scheme devised by Peirce and Siebrits (1996).

The boundary integral equation method (BIEM) is one of the numerical methods most commonly and successfully used in the modeling of crack dynamics, and consists in representing the traction on the crack as a convolution of the slip-rate profile and a certain integration kernel. If the crack is planar in shape, and if we use the piecewise constant (bar graph-like) approximation to discretize the slip-rate and traction profiles on the crack in the numerical modeling in the time domain, the coefficients appearing in the discrete equations of crack dynamics (or influence functions) are given by analytically rigorous forms, so that we can carry out semi-analytic calculations in an efficient way. Moreover, if the ratio of the discrete time step size to the spatial grid size is below a certain value, the equations for different spatial elements decouple, and it becomes possible to obtain the slip-rate at the current time by a simple summation of the past slip history weighted by the influence functions, further enhancing numerical efficiency.

However, the BIEM is prone to numerical instabilities under certain circumstances, in which the numerical output begins to oscillate and finally explodes to infinity as time goes on. It is known that the introduction of an artificial dissipation term or of a slip-weakening friction law can temporarily delay the manifestation of such numerical instabilities, but this does not provide a fundamental remedy. Little is known about the detailed conditions for or the causes of the manifestation of such numerical instabilities, since the mode of their manifestation demonstrates no obvious regularity. Theoretically consistent explanations began to appear only recently, with the advent of the pioneering work by Peirce and Siebrits (1996,97) who dealt with relatively simple cases.

I simulated the time-domain dynamics of a two-dimensional planar crack in all three modes of fracture using the semi-analytic BIEM, and experimentally studied the mode of appearance of the numerical instabilities. It was shown for the first time that the numerical stability had a complicated dependence on the combination of two grid parameters, namely the ratio of the discrete time step size to the spatial grid size, and the relative height of the collocation point along the time axis. In mode II, the numerical scheme is basically unstable unless the set of grid parameters falls in one of the very limited ranges, but it was shown that the scheme could be stabilized by using the epsilon-scheme devised by Peirce and Siebrits (1996). The epsilon-scheme is a technique which consists in placing the collocation point outside the corresponding time step window on the later time side, and accordingly prolonging the time step window corresponding to the current moment.

Inspired by Peirce and Siebrits (1996,97), I also made a preliminary investigation into the relationship between the relative magnitude of influence function components and the numerical instabilities, but I have not been able to identify any obvious correlation between them except in mode III.