We develop a nonlinear wave growth theory of VLF whistler-mode emissions, taking into account the spatial inhomogeneity of the static magnetic field and the plasma density variation along the magnetic field line. We also derive theoretical expressions for the nonlinear growth rate and the amplitude threshold for the generation of self-sustaining rising tone emissions like chorus emissions. The theory is extended for electromagnetic ion cyclotron (EMIC) triggered emissions observed by Cluster spacecraft. We performed a self-consistent particle simulation, in which we inject triggering whistler-mode waves with a constant frequency. The resonant electrons are organized at the resonance velocity in the velocity phase space, and they are released from the triggering wave near the equator. Because of the organized phase structures, the electrons radiate a coherent wave with an increasing frequency that undergoes the nonlinear wave growth due to formation of an electromagnetic electron hole. Self-sustaining emissions become possible when the wave propagates away from the equator. The self-sustaining mechanism can result in rising tone emissions covering the frequency range below the equatorial electron cyclotron frequency. We also performed a hybrid-code simulation injecting EMIC waves as triggering waves. We found the nonlinear wave growth of EMIC triggered emissions below the proton cyclotron frequency. We obtain a pair of coupled differential equations for the wave amplitude and frequency. Solving the equations numerically, we can reproduce characteristics of whistler-mode chorus emissions and EMIC rising tone emissions observed in the inner magnetosphere.

Keywords: wave-particle interaction, whistler-mode wave, chorus, EMIC, nonlinear, radiation belts