

Bifurcation analysis of an extended disk dynamo model incorporating effects of viscosity and variable mutual inductance

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In recent years, a subcritical regime has been found in numerical models of MHD dynamo in a rapidly rotating spherical shell [1]. Under an influence of strong magnetic field, the critical Rayleigh number required for sustaining dynamo action is often lowered. Such kind of dynamo action is classified as subcritical. Subcritical dynamo may be important to understand dynamics and thermal history of planetary cores, such as Mars. However, fundamental physical processes of the subcritical dynamo are not well understood. Moreover, it is not easy to find a basic physics in a three-dimensional MHD dynamo model, which shows spatially and temporally complicated structure. Therefore, it is good as a first step to analyze a simpler model. Here we use a disk dynamo model, extended from the original by Bullard [2] to examine the essentials giving rise to the subcritical dynamo regime.

We extended the Bullard model by taking two effects into account. First, we add a viscous term proportional to the angular velocity of the disk to the equation of motion, where there is no viscous term in case of the Bullard model. Another effect is excitation of different mode of the velocity field modulated by the magnetic field. The mode of the velocity field is varied in a MHD dynamo model, whereas that is fixed to one in the Bullard model. In a MHD dynamo model, dynamo action is likely to be efficient by excitation of different mode of the velocity field modulated by the magnetic field. Therefore, we consider a feedback effect represented by the mutual inductance of the coil, which varies in response to the induced electric current. This effect is added to the equations of motion and dynamo action. We have analyzed the dynamical system governed by the equations with the effects given above. Equations are non-dimensionalized adopting a proper scaling.

We have examined behavior of the extended disk dynamo using a bifurcation theory and numerical simulation. We first investigate the model involving the viscosity with a constant mutual inductance. As a result, it is found that there is a region in parameter space, where dynamo action fails. Then, the model with viscosity and the variable mutual inductance is studied. Consequently, subcritical bifurcation is found in case of two different parameter values describing variability of the mutual inductance. The results suggest that parameter region that can not sustain dynamo action in a MHD dynamo model is due to the viscous force, and that emergence of the subcritical dynamo regime in a MHD dynamo model is due to excitation of different mode of the velocity field modulated by the magnetic field.

References

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Keywords: Dynamo theory, planetary magnetism, subcritical dynamo, disk dynamo