

Interpretation of the G-R law using an arithmetic dynamical system

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The G-R law is known as an empirical law about the magnitude and frequency of earthquakes. In seismic hazard assessment, background seismic activity models that assume the G-R law and a Poisson process have been conventionally used. As critical phenomena, it is well known that earthquakes follow a power law represented by the GR law and phenomenological interpretation have also been made. On the other hand, the attempt to derive the GR law on the basis of dynamical systems is very few.

In order to know the behavior of critical phenomena in the system that a boundary conditions is added, phenomenological interpretation is carried out only for the phenomenon under no boundary condition. If we construct a dynamical system that can produce the G-R law, we expect that we can consider behavior of critical phenomena in systems that boundary conditions are added.

In the following, we consider that we model seismic activity by using the distribution of prime numbers.

Let p_i be i -th prime. $M(p_i) = p_i - p_{i-1}$. Then, $M(p_i)$ can be interpreted as magnitude. The behavior of the number of occurrences $M(p_i)$ follow a power law. A problem that the seismic activity model can be parameterized using a prime number is raised.

On the other hand, in recently, in the field of mathematical physics, as part of the study of noncommutative geometry, so called Bost-Connes system has been studied. As a feature of Bost-Connes system, it is known that the partition function of the system becomes the Riemann zeta-function. By applying the Mellin transform on the Reimann zeta-function, we can obtain the explicit formula on the distribution of prime numbers. By using the explicit formula and zeros of the Riemann zeta-function, the nature of the distribution of prime numbers can be described.

If we can associate a occurrence field of earthquakes with the Bost-Connes system and we can capture the time evolution of the system by variable transformation for the partition function, we may get a dynamical system that can explain seismic activity that produces the G-R law.

References

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Keywords: G-R law, Reimann zeta-function, explicit formula, Bost-Connes system, partition function, prime