

The theory of the unperturbed and perturbed rotational motion of celestial bodies in the Sadov and Kinoshita variables

BARKIN, Mikhail^{1*}, BARKIN, Yury²

¹Moscow Aviation Institute, ²Sternberg Astronomical Institute, Moscow

The analytical theory of the unperturbed and perturbed rotational motion of a rigid celestial body (planet or satellite) is developed. As the unperturbed motion it is accepted and developed the free rotational motion of a rigid triaxial body with unequal principal moments of inertia (the case of the Euler-Poinsot). The body has any triaxiality and arbitrary dynamic oblateness. Also another class of celestial bodies ? non-rigid, and weakly deformed, with a variable geometry of the densities. A new theory of the unperturbed rotational motion of celestial bodies, which takes into account own rotational deformation of the body, is constructed and developed (Chandler-Euler motion, Barkin Yu., 1998). In both cases, the unperturbed rotational motion is a motion on the Euler-Poinsot. However, for weakly deformed celestial body this motion is characterized not by its real moments of inertia, and some the changed constant moments of inertia, taking account of its tidal deformations caused by its own rotation.

Hamiltonian formalism is developed and the unperturbed rotational motion of rigid and weakly deformed celestial bodies are described by <<action-angle>> variables in the form introduced in the well-known works of Yu. Sadov (1972), H. Kinoshita (1977) and Yu. Barkin (1992, 1998). The of paramount importance for the study of the perturbed rotational motion of the planet (satellite) in the gravitational field of the perturbing bodies has the construction of expansions of the force function of the Newtonian interaction of the body with the surrounding celestial bodies in "action-angle" variables. The first attempts to construct such expansions in the theory of rotation of the Earth have been made in the work of H. Kinoshita (1977) for the second harmonic of the force function. In this case the author has made some simplifications and some restrictions on the dynamic oblatenesses of the planet. But it was in the works of Yu Barkin (1992, 1998), these expansions were obtained in an exact representation in the form of Fourier series on multiple variables "angle" with the coefficients represented by functions of the variables "action." This representation of the coefficients is not trivial, and makes extensive use of the apparatus of elliptic functions, theta functions, hyperbolic functions and elliptic integrals of three kinds.

In the papers (Yu. Barkin, 1992, 1998), the expansion of the force function was obtained for the main terms of the second harmonic proportional to the coefficients of the gravitational field of the body C₂₀ (zonal harmonic) and C₂₂ (the main tesseral harmonic). In this paper, a complete expansions in Fourier series of all components of the second harmonic of the gravitational potential, i.e. additional terms proportional to the geopotential coefficients C₂₁, S₂₁ and S₂₂ have been obtained (M. Barkin, 2011). The expansions are presented in a compact form convenient for applications.

Analytical formulas for the first-order perturbations in "action-angle" variables in this more general formulation of the perturbed rotational motion of the planet (the Earth) under the gravitational attraction of external celestial bodies (Moon, Sun) have been obtained in explicit form. We studied also the changeable Earth's rotation with variable coefficients of the gravitational potential C₂₀ (t), C₂₂ (t), C₂₁ (t), S₂₁ (t) and S₂₂ (t). Separately the dynamic effects due to the observed secular changes of these coefficients and the observed annual variations obtained by the long-term observations of geodetic satellites (Cheng et al., 1997, 1999) have been studied.

Follow to modern studies and works of astrophysicists (Link et al., 2001) we developed a number of applications of the formulas of our unperturbed rotational motion to study the kinematics and dynamics of isolated pulsars with precession.

Keywords: Kinoshita variables, Sadov variables, unperturbed and perturbed motion, gravitational attraction, celestial bodies, <<action-angle>> variables

Reconstruction of paleoselenoid using surface shapes of mare basalts and flow directions of sinuous rilles

OGAWA, Tatsuhiko^{1*}, HEKI, Kosuke¹

¹Dept. Nat. Hist. Sci., Hokkaido Univ.

It is believed that the Moon has kept synchronous rotation throughout the history of the Earth-Moon system. However, the current selenoid (geoid of the Moon, an equipotential surface) has imbalances in its degree-2 coefficients as pointed out by Garrick-Bethell et al. (2006). For a synchronously rotating satellite, the ratio of the centrifugal potential to the tidal potential becomes 1:3. Then the ratio between C_{20} ($=-J_2$) and C_{22} should be 10:3 in hydrostatic equilibrium. However, the ratio determined recently with Kaguya is 9.09 (Namiki et al. 2009), that is, J_2 is too large relative to C_{22} .

In order to further study this problem, we try to recover a snapshot of the ancient selenoid during the heavy bombardment period using the surface shapes of lunar maria. We use the results of laser altimeter (LALT) of Kaguya in order to reconstruct the shape of selenoid when mare basalt filled the basins. We will also use the data of the terrain camera (TC) to discuss flow directions of sinuous rilles and difference between the current and past selenoid.

We investigate if the mare surface in a mascon basin is parallel with the present day selenoid by comparing topographic data from LALT (Araki, et al., 2009) and the lunar gravity model (SGM100h, Matsumoto et al., 2010). Because basaltic lava of the Moon has a viscosity lower than any lava flows on the Earth (Murase and McBirney, 1970), its surface might preserve the fossil selenoid when the lava solidified. This was the case for the three mascon basins, Imbrium, Serenitatis, and Humorum, i.e. the LALT data showed subtle curvatures consistent with the selenoid shapes within the basins calculated from the lunar gravity model. In addition to the localized curvatures, selenoid shapes within these basins showed overall tilt relative to the average lunar sphere. The estimated two-dimensional gradients of the Maria Humorum and Serenitatis showed directions consistent with the hydrostatic degree-2 shape (J_2 : $C_{22} = 10:3$), but the Mare Imbrium showed anomalous gradient. Although the Mare Imbrium should have a (upward) slope toward SE under hydrostatic equilibrium, the observed slope was toward SW. In short, the Mare Imbrium seems to have an unexplained westward slope.

In order to confirm this with other evidence, we analyzed the TC topographic data and flow directions of sinuous rilles. Sinuous rille, a channel or valley, is considered to have been formed by thermal erosion of basaltic lava flow. Their flow direction should be downward relative to the current selenoid like rivers on the Earth. R. Nakamura (2011, pers. comm.), however, found an intriguing case that the sinuous rille, Rimae Plato (northeast of Imbrium) seems to have flown upward relative to the current selenoid. This might suggest certain fossilized differences between the present and ancient selenoid. Lots of sinuous rilles exist around the Mare Imbrium, and some sinuous rilles, including Rimae Plato, showed upward flow relative to the current selenoid. Then we found some interesting cases in these sinuous rilles. For example, Rima Suess, which is in the SW of Mare Imbrium, seems to have flown against gravity relative to the current selenoid. However, this flow becomes downward if we correct for the unexplained tilt of Mare Imbrium region assuming that it occurred after the rille formation. So this might support the abnormal tilting of Mare Imbrium. Some sinuous rilles, such as Rima Suess, showed similar signatures, but others did not. So we will systematically survey the flow directions of these rilles to establish a possible scenario of the tilting motion of the Mare Imbrium.

Keywords: Moon, Selenoid, sinuous rille, basaltic lava, Mare Imbrium