The difference between microscopic viscosity and macroscopic viscosity of crystal-bearing magmas

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Magma is a mixture of silicate melts, crystals and bubbles. The amount of crystal and bubble particles in magmas significantly affects its viscosity, which increases dramatically as particle volume fraction increases. Viscosity estimation of magmas is important when we understand the time and space scales of volcanic activities, hence viscosity measurements in analogue experiments and melting experiments were actively conducted. We propose that multi-phase fluids like magmas have two different, apparent viscosities. One is defined as “microscopic viscosity”. It appears when micro objects like crystals move in a magma chamber. The other is defined as “macroscopic viscosity”. It is a bulk viscosity, and appears when a magma rises in the volcanic conduit. In previous analogue experiments for solid-liquid fluids, falling-ball viscometry [1] is considered to measure microscopic viscosity and a rotational viscometer [2] is considered to measure macroscopic viscosity. However, in previous researches, there was no experiment that compares viscosities obtained by these two different methods. Therefore, this study was performed to clarify the differences between microscopic viscosity and macroscopic viscosity by measuring viscosities of one solid-liquid fluid by falling-ball viscometry and with a rotational viscometer.

Material and Experimental Technique: Suspensions of plastic beads of two different radius (0.75mm, 1.5mm with density=930kg/m^3) immersed in corn syrup (Karo corn syrup with density=1400kg/m^3 and viscosity η~7 Pa·s at 23℃) were used as analogues of crystal-bearing magmas. We prepared ten different suspensions by changing particle radius (0.75mm, 1.5mm) and particle volume fractions (F_p=0, 5, 10, 20, 30%). Microscopic viscosity was measured by falling stainless steel balls of three different radius (0.75mm with density=9620kg/m^3, 2.5mm with density=7960kg/m^3, 4.76mm with density=7950kg/m^3) into a 100ml, φ51mm glass beaker filled with the magma analogue. Macroscopic viscosity was measured using a coaxial double cylinder rotational viscometer that is of Kawanami’s own making. We changed the voltage (1.0V, 1.5V, 3.0V) applied to the motor, to investigate the shear thinning behavior.

Results: We used the viscosities measured with a rotational viscometer driven by 1V as representative macroscopic viscosities, because the effect of shere thinning looks low enough. At R_susp=0.75mm, where R_susp is the radius of the suspended particles, the ratio of values of microscopic viscosity to those of macroscopic viscosity, \( \eta_{\text{micro}}/\eta_{\text{macro}} \), were about 0.7-0.9 under the conditions that R_fall/R_susp is 1.0 or 3.3, where R_fall is the radius of the falling ball, and F_p is less than 20%. Moreover, it is suggested that \( \eta_{\text{micro}}/\eta_{\text{macro}} \) is almost 1 under the condition that R_fall/R_susp is 6.4 and F_p is less than 30%. At R_susp=1.5mm, \( \eta_{\text{micro}}/\eta_{\text{macro}} \) is ranging from 0.6 to 0.9 under the conditions that R_fall/R_susp is 0.5, 1.7, or 3.2 and F_p is less than 20%.

Reference
Bubble coalescence in silicate melts: mathematical formulations and experimental observations

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Bubble coalescence deeply affects the dynamics of conduit flow during volcanic eruptions by modifying the rheology of the magma and through the development of structural heterogeneity. To model bubble coalescence in silicate melts, we present a new set of equations that describe the efficiency of the coalescence process as a function of the timescales for diffusive growth and melt-film drainage from bubble-bubble interfaces. The frequency of bubble coalescence is controlled by the timescales of these two processes, which is in turn regulated by the composition and viscosity of the silicate melt. When the vesicularity is less than half, coalescence efficiency varies as a function of the diffusivity of degassing volatiles in melts. At higher vesicularity, the coalescence efficiency is controlled by the melt film drainage. The model predicts an exponential decay of the bubble number density (BND) with time and the exponential bubble size distribution (BSD) function at stagnant conditions, and is in good agreement with in-situ experimental observations of bubble coalescence in basaltic, andesitic and rhyodacitic melt for lower vesicularities. The formulation can be used to estimate an original value of BND formed by a nucleation event using BSDs measured by the textural analysis for pyroclasts which experienced the bubble coalescence. In addition, from values of slopes of approximated BSDs, we can estimate the timescale of magma ascent or the laps time from the onset of bubble coalescence to the quenching. These textural observations for original BNDs and magma ascent timescales allow us to understand roles played by bubble coalescence in controlling the eruption styles and the shifts, using the combined method of geophysical monitoring and modelling.

Keywords: bubble coalescence, BND (bubble number density), BSD (bubble size distribution)
Sawtooth wave-like pressure changes (STW) appeared in a slug flow experiment: Toward understanding of volcanic oscillation systems

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We are developing a laboratory eruption experiment system to investigate multi physics of volcano eruptions. In this study, we focus on a sawtooth wave-like pressure change (STW) observed in a preliminary system that is a syrup eruption experiment. The STW is cyclic pressure changes of which a cycle consists of a gradual pressure increasing stage and an abrupt pressure drop stage. STWs have been observed at many active volcanoes as geodetical signals including tilt, displacement [Genco and Ripepe, 2010; Ohminato et al., 1998].

An apparatus for a slug-flow experiment was designed based on the syrup eruption experiment. This apparatus was equipped with a gas chamber (volume, \( V_c \)) and a vertical pipe for a slug flow. Initially the pipe was partially filled with the syrup to the height of \( H_s \) from the end. Then, gas was injected at a constant mass flux (\( Q_{in} \)) to the chamber to flow into the pipe pushing up the syrup in the pipe. Two representative flow patterns were observed in the pipe. One was characterized by alternate layers of syrup slugs and gas slugs ascending in the pipe, which we called a slug flow. The other was characterized by repetitive transitions between the slug flow and an annular flow, which we called a slug-annular flow oscillation. The STW was observed during the slug-annular flow oscillation.

Pressure change in the chamber and acoustic waves at the vent of the pipe were measured. These measurements were assumed to correspond to geodetic and infrasonic observations at actual active volcanoes. In the experiment, the flow patterns were also constrained by image analyses. The occurrences and features of the STW in the chamber pressure were investigated with taking \( V_c, Q_{in}, \) and \( H_s \) as the experimental parameters. The results showed that the STWs were observed if there were sufficiently large \( V_c \) and \( Q_{in} \), and that the STW changed from periodic to non-periodic cycles with increasing \( Q_{in} \).

A mathematical model was constructed based on the experimental results of the pressure changes and the flow behaviors in the pipe. The model took account of the compressibility of the gas in the chamber, and the nonlinearity of the pressure loss in the pipe flow due to the interaction between the ascending liquid slugs and falling liquid film along the pipe wall. The dependence of the occurrence, the period, and the amplitude of the periodic STW on the experimental parameters were well explained by the model. The model has a mathematically similar aspect compared to existing models for the volcanic oscillation.

Moreover, not only the periodic STW but also the non-periodic STW was observed in this experiment. The non-periodic STW behavior has not been captured by the present model. According to the image analyses, we inferred that the non-periodic behaviors were caused by the interaction between the ascending liquid slugs and surface disturbances of the falling liquid film. From these results, we obtain an insight that irregularity of actual eruptions can be caused not only by fluctuations in ascending flow but also by influences of descending flow such as a fall back, a drain back and a magma convection of magma in the conduit.

Keywords: Volcano, Laboratory experiment, Mathematical model
A dynamical system of conduit flow with magma density change due to gas escape

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In lava dome eruptions, magma viscosity change due to crystallization and magma density change due to gas escape during magma ascent generate positive-feedback mechanisms in conduit flow: as magma discharge rate increases, effective magma viscosity decreases because of delay of crystallization (i.e., reduced viscous wall friction) and magma density increases because of less efficient gas escape (i.e., reduced gravitational load), leading to further increase in the discharge rate. These feedback mechanisms induce complex features of conduit flow such as a cyclic behavior and a drastic change in flow pattern. The effect of magma viscosity change on conduit flow has already been investigated in detail in previous studies based on the modeling of a dynamical system of conduit flow. On the other hand, the effect of magma density change is not well understood, although the importance of this effect has been implied from numerical results of 1-dimensional conduit flow model. In this study, we developed a model for a dynamical system of conduit flow in which magma density change due to gas escape is taken into account, and investigated the effects of the magma density change on conduit flow dynamics.

In our model, flow variables in a cylindrical conduit are spatially averaged in vertical direction, and the conduit is connected with magma chamber surrounded by elastic rocks. The model describes time-series evolutions of magma discharge rate (Q) and pressure at the magma camber (P). In the magma chamber, the time derivative of P (dP/dt) is proportional to the difference between magma influx to the chamber and magma outflux to the conduit (i.e., Q), and its proportionality constant is the parameter $C = G/V_{ch}$ where G is the rigidity of surrounding rocks and $V_{ch}$ is the chamber volume. In the conduit flow, a momentum conservation equation describes the relationship among P, Q, the magma viscosity, and the magma density. In order to take into account the effects of the viscosity and density changes, we calculated the average magma viscosity and density in the conduit under the assumptions of a stepwise increase in the viscosity and a stepwise decrease in the density during magma ascent. The positions of these stepwise changes are determined by the timescale for crystallization ($t_c$) and that for gas escape ($t_g$), and these timescales are controlled by magma properties such as crystal growth rate and magma permeability. The developed model enables us to systematically investigate how the evolutions of P and Q depend on the parameters $C$, $t_c$, and $t_g$.

On the basis of our model, we can obtain the relationship between P and Q in the fixed points (referred to as $P_f$ and $Q_f$) in which the time derivatives of P and Q are equal to 0. The positive-feedback mechanisms by the viscosity and density changes generate a sigmoidal shape in the curve of the $P_f - Q_f$ relationship: the slope of the curve is positive in the low-Q and high-Q regions, whereas it is negative in the intermediate region. In this case, the time-series evolutions of P and Q (i.e., trajectory) show a cyclic behavior when the fixed point in the negative slope is unstable. A notable feature of the effect of the density change on the $P_f - Q_f$ relationship is that the value of $P_f$ in region of the negative slope becomes much lower than the lithostatic pressure. We found that in this case, the magma discharge rate Q reaches 0 during the cyclic behavior in the time-series evolution, which may correspond to the cessation of an eruption. Because whether Q reaches 0 or not depends on the parameters $C$, $t_c$, and $t_g$, we can obtain a critical condition of magmatic and geological parameters for eruption cessation using our model.
Linking petrological and geophysical observations: A case study of the 2011 eruption of Shinmoedake volcano

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Three sub-plinian eruptions were observed during the 2011 eruption of the Shinmoedake volcano, which were well monitored by tiltmeter, GPS, and weather radar (e.g., Shimbori and Fukui 2012; Kozono et al., 2013). To link petrological information to geophysical observations and understand the evolution of magma ascent processes during sub-plinian eruptions, we investigated pumices from these eruptions. At the Nakadake volcano, we observed deposits of the 2011 eruption and collected pumice samples. We primarily investigated gray pumice although two types of pumice (gray and white pumice) were found in the deposits, because this type of pumice reflects major eruptive magma (Tomiya et al., 2013). Two to four pumice lapilli for each subunit were polished, and bulk groundmass and matrix glass compositions were measured. The analytical results showed that the bulk groundmass composition was almost constant for all three sub-plinian eruptions, whereas the composition of the matrix glass changed systematically. Considering that the matrix glass composition reflects the degree of microlite crystallization, we obtained the variation in microlite crystallinity during the three sub-plinian eruptions. The microlite crystallinity decreased from the early stage of the first eruption to the end of the second eruption. The final eruption showed microlite crystallinity similar to that of the first sub-plinian eruption. The porosity obtained from image analyses showed good correlation with microlite crystallinity, i.e., the samples with high and low porosity had low and high microlite crystallinity, respectively. The petrological data above indicate the following scenario. During the first sub-plinian eruption, magma experienced outgassing and microlite crystallization, resulting in the formation of relatively low porosity magma with high microlite crystallinity. The degree of outgassing decreased during the second sub-plinian eruption and the microlite crystallinity decreased. The magma erupted by the final sub-plinian eruption experienced outgassing and crystallization similar to that of the first sub-plinian eruption. The variation in microlite crystallinity can be explained by considering the change in magma decompression rate and/or the change in the final pressure at which the magma is quenched (e.g., Riker et al., 2015).

Linking the petrological and geophysical observations allows us to understand more details of temporal evolution of explosive eruptions. Geodetic data indicated that the magma fluxes were almost constant during the three sub-plinian eruptions, whereas the pressure in the magma chamber monotonically decreased corresponding to the eruptions (Kozono et al., 2013). These observations are counterintuitive because it is commonly expected that the flux decreases in response to the decrease in the pressure of the magma chamber under the assumption of magma chamber of constant volume. However, these paradoxical observations (at least those from the first and second sub-plinian eruption) may be qualitatively explained by considering that magma fragmentation pressure increased, as recorded in the groundmass of pumices, i.e., the decrease in microlite crystallinity observed from the first to the second sub-plinian eruption. According to the steady conduit flow model (Kozono and Koyaguchi, 2009; Koyaguchi, 2016), even when the magma chamber pressure decreases, the magma flux can be kept constant if the fragmentation pressure slightly increases so that the length of gas-pyroclastic flow regime in the conduit increases, i.e., the
level of the fragmentation surface descends.

Keywords: Eruption dynamics, Magma fragmentation, Petrological and geophysical observations
AD 2015 eruptive activity induced by basalt input at Sakurajima volcano: Inference from petrological monitoring data

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Sakurajima volcano, located in southern Kyushu, Japan, has resumed its eruptive activity at Showa crater since June 2006. After that, the volcanic explosions has been continued until 2013. Although the number of explosions declined in 2014, the volcano had become higher level of eruptive activity accompanied with clear inflation since January 2015. In addition, the dyke intrusion event had occurred in August 2015. In order to reveal the magma plumbing system of the 2015 eruptive events, we carried out the petrological examination of the 2015 juvenile lapilli. We also discussed the possible reason for the activation of the 2015 eruptive activity.

The 2015 juvenile lapilli consists of lithic, scoria, pumice, and a small amount of altered rocks. Plagioclase, orthopyroxene, clinopyroxene, and Fe-Ti oxides are the dominant phenocryst phases, and a small amount of olivine phenocrysts are often occurred. The core compositions of the olivine phenocrysts without reaction rim are Fo80-81, compositionally disequilibrium with the co-existed pyroxenes. On whole-rock chemistry, the 2015 juveniles range 58.3-59.0 wt.% in SiO₂, exhibiting the most mafic compositions in all the samples since 2006. These samples are plotted on the same compositional trends of the juvenile materials since 2006 on Harker diagrams. On matrix glass chemistry, the 2015 juveniles show clearly lower in SiO₂ than those of the activity before 2014. In addition, within 2015, the silica content becomes lower with time.

The similar petrological features to the juveniles since 2006 as well as the consistency in the compositional trends of whole-rock chemistry suggest that the magma plumbing system since 2006 has been continued in 2015. The most mafic compositions of the 2015 juveniles both in whole-rock and matrix glass chemistries reflect that the considerable input of basaltic magma had occurred since 2015. Comparing to the geophysical monitoring data since January 2015, as the ratio of the basaltic magma in erupted magma increased, the volcanic edifice inflated and the eruptive activity became larger. After that, the activity changed to the magma intrusion event. It is highly probable that the activation of the 2015 eruptive activity had been induced by newly input of basaltic magma.

Keywords: Sakurajima volcano, glass chemistry, whole-rock chemistry, temporal change
Mathematical formulation of forecasting volcanic eruption sequence based on physical models and field observations

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During a volcanic eruption, the intensity and style of eruption generally change with time. In order to establish a method to forecast such eruption sequences, forward and inverse problems are mathematically formulated on the basis of physical models for the magma plumbing system including the conduit flow dynamics and the magma chamber processes.

The variation of eruption sequence is characterized by how magma discharge rate, Q, changes with time as a function of magma chamber pressure, P. According to the conduit flow models, the qualitative feature of the relationship between the magma discharge rate and the chamber pressure (the Q-P relationship) during explosive eruptions is controlled by the pressure at which the conduit flow changes from a bubbly flow to a gas-pyroclast flow (i.e., the fragmentation pressure). The fragmentation pressure, in turn, depends on the mechanisms of gas-escape and magma fragmentation. The Q-P relationship during non-explosive (effusive) eruptions depends on the density change due to gas-escape process and the viscosity change due to crystallization during magma ascent in the conduit. The physical model of magma chamber processes, on the other hand, suggests that the Q-P relationship is affected by the effective compressibility and volume of magma chamber. The effective compressibility of magma chamber drastically increases when the magma contains gas phase, and hence, it depends on water content and pressure of magma. Because of the coupled effects of conduit flow dynamics and magma chamber processes, the forward model of the magma plumbing system shows diverse behavior of eruption sequences (i.e., various trajectories of the Q-P relationship).

In order to forecast the eruption sequence, we must estimate the values of parameters that control the trajectories of the Q-P relationship in the above forward model. Generally, the inverse problem of the magma plumbing system is formulated as a problem to estimate the product of volume and pressure change of magma chamber and the effective compressibility of magma chamber from the field data on ground deformation around the volcano and mass of the erupted magma. The estimation of the rest of the parameters (e.g., the density and the viscosity of magma in the conduit) requires additional field observations such as petrological data of erupted magma. The numbers and kinds of parameters that can be estimated from the inverse model depend on the mathematical characteristics of the conduit flow model. For effusive eruptions where the conduit flow is approximated by a Poiseuille flow so that P and Q are proportional, a parameter expressed by the combination of viscosity, conduit length, conduit radius, chamber volume and effective compressibility of magma chamber is collectively estimated from the decay constant of Q and P during the waning stage of the eruption. For a certain type of explosive eruptions, on the other hand, the value of the fragmentation pressure can be constrained by the trajectory of the Q-P relationship observed during the waning stage of the eruption. In the presentation, we mainly discuss how the uncertainties of the parameter estimation and the forecast of eruption sequence depend on the mathematical characteristics of the conduit flow model.

Keywords: volcanic eruption sequence, physical model, magma plumbing system
Comparison between geodetic data and volcanic conduit flow models

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Volcanic conduit flow models have been improved from many results of geologic sample analyses and laboratory experiments, and have been used for understanding the behaviors of magma flow in the conduit based on the spatio-temporal changes of pressure and velocity etc. of volcanic flow that are numerically and analytically obtained. Since the geodetic data can quantify locations and magnitude as well as shapes of the pressure sources in the volcanic edifices, analyses of geodetic data are quite useful to directly examine the volcanic flows in real volcanoes. The present study summarizes and discusses relations between geodetic data analyses and volcanic flow model, giving attentions into gas bubble growth and rising, and out-gassing during magma ascent as well as propagation of magma-fragmentation surface in the conduit during eruptions.

Keywords: Volcanic conduit flow model, geodetic data
Lava dome eruption: Sinabung (Indonesia) vs. Unzen (Japan)

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Processes of lava dome eruptions between Sinabung and Unzen volcanoes are similar to each other; they started with phreatic eruptions which advanced through phreatomagmatic eruption to magmatic eruptions. Lava collapsed-type pyroclastic flows had repeated during the growth of the lava dome and/or flow at both volcanoes. Before the appearance of the lava in the summit crater, inflation of the volcanic bodies was observed, and, during the growth of lava dome/flow, deflation of the volcanic bodies continued with the extent decreased with time. Lava effusion rates which peaked with about 6 m³/s decreased with time at the both volcanoes. Lava of Sinabung is hornblende andesite (>900 °C), while that of Unzen is hornblende-biotite dacite (<850°C). The melts are high-silica rhyolite, and their compositions were controlled by effusion rate. At Sinabung, the precursory phreatomagmatic eruptions were vulcanian and the latest stage is characterized by repetition of small vulcanian events. Lava dome extended into a lava flow as long as 3 km long. In conclusion, the lava effusion rate change controlled the pattern of lava dome growth, and the difference of melt temperature may have controlled the explosivity and the length of lava dome/flow.

Keywords: Lava dome eruption, Unzen volcano, Sinabung volcano
Understanding of caldera-forming eruption from geological and petrological approaches

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Decompression process prior to caldera collapse is one of the key processes for caldera-forming eruption. In many caldera-forming pyroclastic eruptions, precursory eruption decompresses magma chamber and consequently induces the faulting and subsidence of the roof of magma chamber. The eruption of main ignimbrite follows the onset of collapse. Formation of collapse caldera indicates that the magma pressure within a magma chamber drops below the threshold for collapse. Exposing internal structure in many eroded calderas and drilling into a young collapse calderas reveals that the collapse calderas are filled with thick intracaldera ignimbrite more than 1 km in thickness. Existence of such a thick deposit inside collapse caldera strongly suggests that the caldera collapse is simultaneous with the eruption of main ignimbrite. Many large ignimbrites are preceded by smaller pyroclastic eruption. Such precursory eruption can be a large Plinian eruption, smaller ignimbrite, or combination of both. These precursory eruptions withdraw magma from magma chamber to decompress the magmatic pressure within the chamber. The decompression reached to a threshold for collapse when the end of the precursory eruption. Petrological evaluation of decompression within a magma chamber prior to the onset of collapse is crucial to understand the trigger for the main ignimbrite.

Keywords: large-scale eruption, caldera, magma
The possibility of rapid and huge magma accumulation in the crust from dynamical point of view

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As much as 100s-1000s km$^3$ magma eruption in a single event (Machida & Arai, 1992) proves huge magma accumulation in the crust before eruption. Moreover, Takada (1999) indicates several times as much as erupted magmas may accumulate in the crust from ratio of erupted volume to accumulated volume. Although the magma accumulation rate for caldera eruption can be calculated to be 0.001-0.01 km$^3$/year on average (Salisbury et al., 2011), its accumulation process has not clearly understood yet (Jellinek & DePaolo, 2003).

Druitt et al. (2012) examined composition of some plagioclases from Santorini volcano which emitted 40-60 km$^3$ of magma and concluded that a few km$^3$ magma were added to a magma chamber in about 100 years. This rapid magma accumulation rate is about 0.01-0.1 km$^3$/year, ten times as large as foregoing one. This result can be crucial for volcanic eruption prediction because the accumulation may cause large scale crustal deformation. However, this petrological result has not been examined whether it also meets the dynamic constraint or not. In order to clarify this point, our study intends to estimate the maximum magma volume that the crust can accumulate in short time by using FEM (Marc Mentat). The crust is assumed as an elastic body since about 100 years is relatively short time compared with Maxwell relaxation time of the crust.

In our analysis, we inflate the magma chamber by pressuring chamber wall and compared the resulted strain around it with the ultimate strain of the crust $10^{-4}$-$10^{-5}$ (Rikitake, 1975). Our hypothesis is that two of the influential parameters involving large magma accumulation may be magma chamber shape and a magma chamber volume that has already existed before a new magma is added (hereinafter called, “primary volume”). Therefore, the calculation was carried out for spherical magma chamber and spheroid-shaped sill which have 100-2000 km$^3$ of primary volume, respectively. The upper depth of magma chambers are fixed at 5 km depth (Yasuda et al., 2015); that is, the central depth of these chamber are different between models. We assumed that the surface of the Earth to be free surface, the crust to be isotropic and homogeneous, $\lambda = \mu = 40$ GPa (Mogi, 1957), and ignored the gravitational effect. In addition to this numerical calculation, we also computed two analytical formulae as a reference, Mogi model (Mogi, 1958) for spherical chamber and tensile fault model (Okada, 1992) for sill, under the same condition. Note that these models are only applicable on the condition that primary chamber volume are very small.

As a result, maximum shear strain exponentially decreased as primary volumes increase in both types of chambers, and the maximum value was obtained at the analytical solution. Fig.a,b shows the maximum shear strain on the surface caused by an expansion of magma chamber which has 2000 km$^3$ of primary volume. For both models, volume increment was proportional to the maximum shear strain, while sill had smaller intercept for same volume increment. This result means that sill-shaped magma chamber has larger potential for magma accumulation than spherical chamber when same volume of magma accumulates. However, even the primary volume is as large as 2000 km$^3$, the strain derived from more than 1 km$^3$ volume increment exceeds the ultimate strain of the crust. Generally, when a strain is bigger than the ultimate strain, the crust cannot be dealt as an elastic body because the crust around magma chamber yields or causes brittle fracture; that is, we think that the discussion which considered plastic deformation or brittle fracture is necessary when we illustrate the crustal deformation in case that a few km$^3$ of magma accumulate in about 100 year which Druitt et
al. 2012 proposed, regardless of the difference of magma chamber shape or variety of primary volume.

Keywords: large volcanic eruption, magma accumulation, crust, strain, stress, caldera

![Diagram showing the maximum shear strain in the surface versus volume increment for a spherical magma chamber and a spheroidal sill.](image)

Fig. a, b: The maximum shear strain in the surface versus volume increment for a. Spherical magma chamber (radius 7.8 km, central depth 12.8 km, upper depth 5 km) and b. Spheroidal sill (aspect ratio 10:10:1, major radius 16.6 km, minor radius 1.68 km, central depth 6.68 km, upper depth 5 km). Primary volume is 2000 km$^3$, respectively. The distance starts and radially directed from immediately above the chamber in the surface. The shaded region shows the probable range of the ultimate strain of the crust and the region to the right of the dash line indicates the rapid magma accumulation volume by Druitt et al. (2012).