Truncated SVD methods for linear ill-posed problems

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have been widely used in inversion. How to truncate determines the quality of a truncated SVD solution, but truncation was often done arbitrarily. The first workable criterion for truncation was based on F-statistical testing but almost not used in geophysical inversion. Recently,
the L-curve approach was proposed for the same purpose and soon found many applications in interdisciplinary inverse problems. Up to the present, very little has been known of the solution quality by these two workable criteria. We have thus proposed in this paper a first, new quality-based method for truncation.

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the L-curve approach was proposed for the same purpose and soon found many applications in interdisciplinary inverse problems. Up to the present, very little has been known of the solution quality by these two workable criteria. We have thus proposed in this paper a first, new quality-based method for truncation. Six TSVD estimators have been investigated for comparison in regularizing discrete unstably ill-posed problems, based on the statistically frequently used {\em
F}-statistic, the L-curve of much recent interest, and our new quality-based MSE criterion. The three F-statistic-based TSVD estimators tested can indeed marginally improve the LS solution to the ill-posed downward continuation problem in the sense of long term averaging, depending on pre-selected
significance levels. The simulations have shown that the estimators of this type can hardly guarantee the improvement of the condition number of the linearized unstably ill-posed system. In other words, F-statistic
criterion can frequently lead to wrong cuttings of components. The TSVD
estimator by means of L-curve is the
best in (over)stabilizing ill-posed problems but results in an over-cutting of components. It is extremely poor in terms of biases and MSE roots of the
solution. The quality-based TSVD estimator with the basic ridge estimate of X as its initial values has outperformed the other two TSVD techniques tested in terms of solution stability, bias and mean squared error. The simulations have clearly shown significant quality advantages of the new method. The simulations have also indicated that the new method is able to achieve a mean 5 mgal accuracy of gravity anomalies from satellite gradiometry, if a few biggest biases are left out of the computation.