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Non-hypersingular boundary integral equations for 2-D and 3-D crack dynamics

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We present, in a fully explicit form, a non-hypersingular BIE for the dynamic time-domain analysis of the 3-D non-planar crack. One of the key concepts in our study is that of a local Cartesian coordinate system, one of whose axes is always held locally perpendicular to the potentially curved surface of the crack. Later in the talk, we detail on a highly efficient numerical method for solving the dynamics of the planar 2-D crack based on the non-hypersingular BIEM. The method, valid only for planar cracks, consists in applying a piecewise-constant interpolation to the slip-rate and rewriting the BIE into a time-marching scheme of discrete equations with analytically obtained convolution coefficients. More specifically, we address the problem of numerical stability.

The boundary integral equation method (BIEM) is one of the most viable approaches to deal with crack-analysis problems, which are widely dealt with in seismological research.

According to the displacement discontinuity strategy, the stress field over the model space is expressed as a convolution, in time and space, of the slip along the crack and a set of integration kernels. Then a limiting process is so applied that the receiver point may approach the crack face, producing a set of boundary integral equations (BIEs) that relate the traction on the crack surface to the slip on it. The traction BIEs, thus derived, are hypersingular, and are not immediately amenable to numerical implementation. One way to circumvent this is the regularization approach, which consists in rewriting, most often through integration by parts, the hypersingular integrals in an equivalent form which involves only weakly singular integrals, at most integrable in the sense of Cauchy principal values.

We present here our recent developments in the time-domain BIEM theory for crack dynamics which is based on the above-described approach. The present talk consists of two parts.

In the first part, we demonstrate how we have derived, for the first time to our knowledge, a non-hypersingular BIE for the 3-D non-planar crack in the time domain and in a fully explicit form. One of the key concepts in our study is that of a local Cartesian coordinate system, one of whose axes is always held locally perpendicular to the potentially curved surface of the crack. Its use allows us to formulate the BIE in terms of the one opening and two shear components of the slip on the crack surface. Similar expressions have also been derived for the stress and displacement field outside the crack. Our new equations, complete for cracks of arbitrary geometry, are consistent with existing theories on the 3-D planar crack (Fukuyama and Madariaga, 1998) and on the 2-D non-planar crack (Tada and Yamashita, 1997).

In the second part, we detail on a highly efficient numerical method for solving the dynamics of the planar 2-D crack based on the non-hypersingular BIEM. The method, used earlier by Cochard and Madariaga (1994) for the 2-D anti-plane (mode III) crack and by Fukuyama and Madariaga (1998) for the 3-D crack, is valid only for planar cracks and consists in applying a piecewise-constant interpolation to the slip-rate and rewriting the BIE into a time-marching scheme of discrete equations with analytically obtained convolution coefficients. We apply this numerical method to the case of the planar 2-D in-plane (mode II/I) crack, and also show that a similar numerical scheme can be used to calculate the stress and displacement field off the crack plane.

More specifically, we address the problem of numerical stability. We executed an exhaustive set of trial runs, with different combinations of two control parameters, of which one concerns the ratio of the temporal grid size to the spatial grid size, and the other concerns the position along the temporal axis within each spatiotemporal grid of the point where collocation is carried out in the discretization procedure. We found out that, at least for the case investigated, the scheme was highly stable over a wide range of parameters in modes III and I, whereas, in mode II, it was unstable for the most part of the parameter space but was stable for some very limited ranges of the parameters.