Sb-P006　　会場：IR　　時間：6 月 26 日　17:30-19:00

群遅延時間の分散のスペクトルによる位相のモデル化（その１０）ＦＦＴを用いた群遅延時間の計算法

Modeling of phase using variance spectrum of group delay time, Part 10:A new method to calculate group delay using FFT

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地震動のフーリエ位相スペクトル特性を表すのに，群遅延時間が用いられることが多くなってきているが，これの計算には，通常，直接数値微分を用いられてきた．しかしながら，この方法では，元の時刻歴の後ろに付けるゼロの数を変化させると，求められる群遅延時間の値が変化する，計算の前に人工的なアンラップ操作が必要であるなどの欠点があった．そこで本研究ではＦＦＴを用いた計算法を提案する．この方法によれば，アンラップ操作をすることなしに，後続のゼロの数を変えても同一の群遅延時間が求められる．

1. Introduction

Group delay time is known as a good parameter to represent characteristics of Fourier phase spectrum of non-stationary time histories. We propose a new method to calculate group delay time without artificial un-lapping and direct numerical differentiation.

2. Group Delay Time

Let $f(t)$ be a non-stationary time history and $F(\omega)$ the Fourier transform of $f(t)$;

$$F(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}dt = A(\omega)e^{-i\phi(\omega)} \quad , \tag{1}$$

where $i=\sqrt{-1}$. $A(\omega)$ is the Fourier amplitude spectrum and $\phi(\omega)$ the Fourier phase spectrum. Group delay time (tgr) is defined as the gradient of the phase spectrum with respect to the frequency, ω. The finite difference approximation is conventionally used for numerical calculation as

$$tgr(\omega)=\frac{d\phi(\omega)}{d\omega}\approx\frac{\phi(\omega+\Delta\omega)-\phi(\omega)}{\Delta\omega} \quad . \tag{2}$$

An artificial un-lapping process is necessary to calculate the above equation.

3. Proposed Method

We propose a new method to calculate $tgr(\omega)$ numerically using FFT as follows;

$$\begin{aligned} T_{gr}(\omega)&=\frac{d\phi(\omega)}{d\omega}=\frac{d}{d\omega}\left\{-\tan^{-1}\frac{F_I(\omega)}{F_R(\omega)}\right\} \\ &=\frac{F_I(\omega)\cdot\dfrac{dF_R(\omega)}{d\omega}-F_R(\omega)\cdot\dfrac{dF_I(\omega)}{d\omega}}{F_R{}^2(\omega)+F_I{}^2(\omega)} \end{aligned} \quad . \tag{3}$$

where $F_R(\omega)$ and $F_I(\omega)$ are the real and imaginary part of $F(\omega)$. Here $g(t)$ is defined by Eq.(4). Its Fourier transform $G(w)$ is given by Eq.(5).

$$g(t)=-i\cdot t\cdot f(t) \tag{4}$$

$$G(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} g(t)\, e^{-i\omega t}dt \tag{5}$$

The real and imaginary parts of $G(w)$ therefore are represented by

$$G_R(w) = \frac{dF_R(w)}{dw} \quad , \tag{6}$$

$$G_I(w) = \frac{dF_I(w)}{dw} \quad . \tag{7}$$

$tgr(w)$ is obtained by substituting the Fourier transforms of $f(t)$ and $g(t)$ into Eq.(3).

4. Advantage of the proposed method

The conventional method gives a different result if additional zero trains are included after the time history $f(t)$. The proposed method gives the same values at the discrete frequencies which exist in the Fourier transform of original time history.