Sb-P006 Room: IR Time: June 26 17:30-19:00

Modeling of phase using variance spectrum of group delay time, Part 10:A new method to calculate group delay using FFT

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Group delay time is often used for representing the characteristics of Fourier phase spectrum of seismic motion. Direct numerical differentiation method is conventionally adopted for calculation of group delay time. However this method gives the different value if the zero trains are included after the original time history. We propose a new method to calculate group delay time using FFT. The proposed method gives the same values at the discrete frequencies which exist in the Fourier transform of original time history without an artificial un-lapping process.

1. Introduction

Group delay time is known as a good parameter to represent characteristics of Fourier phase spectrum of non-stationary time histories. We propose a new method to calculate group delay time without artificial un-lapping and direct numerical differentiation.

2. Group Delay Time

Let $f(t)$ be a non-stationary time history and $F(\omega)$ the Fourier transform of $f(t)$;

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\, e^{-i\omega t} dt = A(\omega) e^{-i\phi(\omega)} \quad , \tag{1}$$

where $i = \sqrt{-1}$. $A(\omega)$ is the Fourier amplitude spectrum and $\phi(\omega)$ the Fourier phase spectrum. Group delay time (*tgr*) is defined as the gradient of the phase spectrum with respect to the frequency, ω. The finite difference approximation is conventionally used for numerical calculation as

$$tgr(\omega) = \frac{d\phi(\omega)}{d\omega} \approx \frac{\phi(\omega+\Delta\omega) - \phi(\omega)}{\Delta\omega} \quad . \tag{2}$$

An artificial un-lapping process is necessary to calculate the above equation.

3. Proposed Method

We propose a new method to calculate $tgr(\omega)$ numerically using FFT as follows;

$$\begin{aligned} T_{gr}(\omega) &= \frac{d\phi(\omega)}{d\omega} = \frac{d}{d\omega}\left\{-\tan^{-1}\frac{F_I(\omega)}{F_R(\omega)}\right\} \\ &= \frac{F_I(\omega)\cdot\dfrac{dF_R(\omega)}{d\omega} - F_R(\omega)\cdot\dfrac{dF_I(\omega)}{d\omega}}{F_R{}^2(\omega) + F_I{}^2(\omega)} \quad . \end{aligned} \tag{3}$$

where $F_R(\omega)$ and $F_I(\omega)$ are the real and imaginary part of $F(\omega)$. Here $g(t)$ is defined by Eq.(4). Its Fourier transform $G(w)$ is given by Eq.(5).

$$g(t) = -i\cdot t\cdot f(t) \tag{4}$$

$$G(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(t)\, e^{-i\omega t} dt \tag{5}$$

The real and imaginary parts of $G(w)$ therefore are represented by

$$G_R(\omega) = \frac{dF_R(\omega)}{d\omega} \quad , \tag{6}$$

$$G_I(\omega) = \frac{dF_I(\omega)}{d\omega} \quad . \tag{7}$$

$tgr(w)$ is obtained by substituting the Fourier transforms of $f(t)$ and $g(t)$ into Eq.(3).

4. Advantage of the proposed method

The conventional method gives a different result if additional zero trains are included after the time history $f(t)$. The proposed method gives the same values at the discrete frequencies which exist in the Fourier transform of original time history.