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Topology in Plasma Physics

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The nonlinear dynamics of plasmas are characterized by some conservation laws that are related with topological constraints on electromagnetic fields and flows. This paper is intended to provide an idea of how a mathematical theory of topology can grasp the structure created in plasmas. The Beltrami fields, eigenfunctions of the curl operator, represent essential characteristics of twisted, spiral, chiral or helical structures in various vector fields. Amongst diverse applications of the theory of Beltrami fields, the present paper focuses on the self-organized states of plasmas.

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The Beltrami fields, eigenfunctions of the curl operator [1], represent essential characteristics of twisted, spiral, chiral or helical structures in various vector fields. Amongst diverse applications of the theory of Beltrami fields, the present paper focuses on the self-organized states of plasmas. The helicity, which is a constant of motion in various vortex dynamics systems, represents the linking of field lines, and determines the structural parameters of Beltrami fields.

The Taylor relaxed state is the principal example of self-organized Beltrami fields. When we drive a current and sustain the total helicity, the plasma relaxes into the Taylor state and achieves the Beltrami magnetic field. When a strong flow is implemented to a plasma, self-organized states becomes qualitatively different from the conventional relaxed stats. The two-fluid effect induces a coupling among the flow, magnetic field, electric field and the pressure, resulting in a "singular perturbation" to the MHD system [2]. To invoke this effect, one must supply a driving force to sustain a strong flow. It is equivalent to giving an internal electric field or applying a steep gradient in pressure, because these fields are tightly coupled.

In the two-fluid model, the Beltrami condition demands that the vorticity parallels the flow in both electron and ion fluids. We find that a superposition of two Beltrami magnetic fields (and also two Beltrami flows) solves the simultaneous two-fluid Beltrami conditions. Despite this simple mathematical structure, the set of solutions contains field configurations that are far richer than the conventional theory. The hydrodynamic pressure of a shear flow yields a diamagnetic state that is suitable for confining a high-beta plasma. The H-mode boundary layer is an example, which is spontaneously generated by the core plasma pressure [3]. Active control of shear flow will significantly extend the scope of such self-organized states.

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