A numerical simulator for electromagnetic induction in a 3-D sphere by Finite Element Method using Edge-Element

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To map global electrical conductivity heterogeneities, we need a numerical simulator as a forward solver for electromagnetic (EM) induction in a 3-D sphere. Recently, several numerical forward solvers have been devised. We developed a new 3-D simulation code independently, in a different way. A finite element method with the formulation of magnetic vector potential is presented for the EM induction in a 3-D conducting sphere. In our simulator, the edge-element vector shape function is used on each edge of a tetrahedral cell.To evaluate the code developed in this study, we validate our simulator with the earlier reported quasi-analytical solutions and with the other numerical codes.

Recent studies on global and regional seismic tomography clearly indicated a large-scale heterogeneity in the Earth's mantle. This has also been revealed by electrical conductivity structure obtained by electromagnetic induction studies. To map such large-scale electrical conductivity heterogeneities, we need a numerical simulator as a forward solver for electromagnetic induction in a 3-D sphere. Recently, several numerical forward solvers for global electromagnetic induction in a heterogeneous sphere have been devised viz., the perturbation expansion method, the node-based finite element method , the finite difference method, the spectral-finite element method, etc. The difficulty in developing a 3-D simulator, however, is an evaluation of it's solution toward a realistic complex 3-D sub-surface structure. We developed a new 3-D simulation code independently, in a different way. A finite element method with the formulation of magnetic vector potential is presented for the electromagnetic induction in a 3-D conducting sphere excited by external source current. In our simulator, the edge-element vector shape function is used on each edge of a tetrahedral cell. This method strictly satisfies the discontinuity of the normal electric field across the conductivity boundaries without considering the enforced normal boundary conditions that is usually practiced in some numerical solvers. The use of the edge-element method is more efficient compared to the node-based element method, because in this method, the magnetic vector potential is solved only as a single vector, whose direction is tangential to the edges of the elements. To evaluate the code developed in this study, we validate our simulator with the earlier reported analytical and quasi-analytical solutions and with the other numerical codes.