

## Solving three-dimensional crack dynamics problems with a semi-analytic boundary integral equation method using triangular elements

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In solving 3-D crack dynamics problems numerically in the time domain using the boundary integral equation method, it is possible to carry out calculations efficiently and with high precision, provided that we impose some simplifying conditions on the geometry of the discrete elements and on the distribution of slip thereon so that the rigorous form of the discretized integration kernels (Green's functions) can be obtained analytically. Such a semi-analytic modeling method, using rectangular elements, has already been proposed, but a patchwork of rectangles can represent only a limited class of 3-D crack geometries. I propose a similar semi-analytic method of numerical modeling using triangular elements, so that a curved crack of arbitrary shape can be given an approximate representation.

The boundary integral equation method (BIEM) is one of the numerical modeling techniques used to solve crack dynamics problems, in which traction on the crack surface is represented as a convolution of the distribution of slip (or slip-rate) with certain integration kernels (Green's functions). Most commonly, slip on a crack surface that occurs in response to a given distribution of traction is computed by solving this integral equation. In the calculation, the crack surface is discretized into a patchwork of small constitutive elements, and the integration kernels are also correspondingly discretized into a set of coefficients representing the traction on a certain discrete element in response to a certain mode of slip taking place on another discrete element. Time is also discretized in the case of calculation in the time domain.

If, for an infinite, homogeneous and isotropic medium, we assume that the discrete crack elements have simple and planar geometries and that slip-rate is distributed uniformly in both space and time on each crack element and within each discrete time interval, the convolution coefficients in the discretized equation to be solved can be derived analytically in a rigorous form, making it possible to carry out "semi-analytic" computations efficiently and with high precision.

Such a semi-analytic method of modeling, based on the use of rigorous convolution coefficients, has already been proposed and established for three-dimensional (3-D) crack dynamics problems in the time domain, in which square or rectangular discrete elements are used (Fukuyama and Madariaga, 1998; Aochi, Fukuyama and Madariaga, 2000). However, a patchwork of rectangular elements alone is capable of representing only a fairly limited class of 3-D crack geometries. It can represent a curved crack, so to speak of a cake roll shape, which always displays an identical section no matter at which location it may be sliced along a certain direction, but it is generally impossible, with rectangular elements alone, to represent curved crack surfaces of more complicated geometry.

In order to make up for this weakness in the technique, I define and propose a method for the numerical modeling of 3-D crack dynamics with a semi-analytic BIEM using discrete crack elements of triangular shape. In principle, a patchwork of triangular elements is capable of giving an approximate representation to curved cracks of any arbitrary shape. I obtained analytically the rigorous form of the coefficient of traction on a certain discrete element in response to slip taking place on another discrete element, on the assumption that slip-rate is distributed uniformly in both space and time on each triangular element and within each discrete time interval. The calculation procedure turned out to be more complicated than for rectangular elements. My current presentation will be centered on outlining the above-mentioned theoretical framework and the expression of the analytically obtained response coefficients, but I would also like to report on the latest state of the endeavors toward their application to the numerical simulation of crack dynamics.