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A new method of spherical harmonic analysis

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Spherical harmonic analysis is widely used in broad area of Earth sciences. In particular, expressions using spherical harmonic functions are often used in such simulations as mantle convection, general circulation model, and geodynamo, becuse of their ability to fit easily to globally defined boundary conditions. In such nonlinear problems, it is impracticable to stay in the spectral space all the time, since the necessary computations become enormous. Accordingly, the so-called pseudo-spectral method is often used, in which nonlinear interactions are evaluated in the physical space, and the result transferred to the spectral space to compute the next time step. To move from the physical space to the spectral space and its inverse operation, we employ spherical harmonic transform, which really is the combination of Fourier transform in longitude and Legendre transform in latitude. The problem with this method is that the former can be done by fast method (FFT), while there is no such fast method for the latter. Consequently, most of the CPU time in simulation is taken up by the Legendre transform. A number of attempts were reported to speed up the transform, but none was successful so far.

In this paper, I show that the fast transform is possible because of the following properties of the Legendre functions.

(1) A function defined on a sphere can be divided by the azimuthal wave number m, by first applying the Fourier transform in longitude.

(2) The Legendre series belonging to order number m can be represented by a product of $\sin^m\theta$ and a degree $L-m$ Fourier cosine series (L is the truncation level).

(3) Thus the Legendre series corresponding to m can also be described by a degree L Fourier cosine series (when m is even) or sine series (m is odd).

(4) There are fast computation methods for the transformation from (2) to (3), and its inverse (without the need for multiplication).

By these properties, it can be shown that the fast computation of spherical harmonic transform is indeed possible. Actually, however, the transformations from (2) to (3) and from (3) to (2) are delicate in that they tend to expand errors enormously. It was found that double precision (64 bits) is not enough for meaningful calculations. In addition, in contrast to the case of ordinary Fourier transform, the forward and inverse transforms cannot be implemented by a same procedure. It follows that we need to overcome some further problems in order to use the fast transform method for the actual simulation studies.