H060-011 Room: C304 Time: May 29 11:33-11:45

A new model in groundwater hydrology: simulational study of advection-dispersion equation with fractional derivative

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As the governing equation of the transport phenomenon of contaminants in groundwater, the advection-dispersion equation(ADE) is widely used. However, it has been recognized gradually that ADE does not reproduce some of the experimental results in heterogeneous porous media. Therefore, in this study, we introduce a new governing equation where the heterogeneity of geologic media is taken into consideration, based on Benson et al.'s(1998) research. In this equation, the order of the derivatives is non-integer (i.e. the 0.8th in terms of time or the 1.5th in terms of space), instead of dC/dt or d^2C/dx^2 in the normal advection-dispersion equation. Such an advection-dispersion equations is called as the fractional advection-dispersion equation(fADE). It is obtained as follows.

In the one-dimension random walk model, when a particle jump width dx and waiting time until the next jump dt are set constant, the normal advection-dispersion equation is derived. Here, the jump width dx is assumed to have distribution in consideration of the heterogeneity of a media. The distribution function is assumed to have a form where the central limit theorem does not hold. By carrying out Fourier transform of the number density with such a distribution of dx, the governing equation with non-integer derivative is obtained which is order of alpha in space.

Furthermore, in consideration of the influence of adsorption to the geology media of contaminants, we assume the waiting time dt should have distribution. Choosing the distribution function of dt the same as above, a governing equation with non-integer derivative which is gamma order in time is obtained.

To summarise, the fractional derivative in space is the result of the heterogeneity of flow in a medium whereas the fractional derivative in time is result of the heterogeneity of the adsorption time.

We should numerically the fractional governing equation by the finite difference method, and compare with experimental data. In deifferentiating the fractional derivatives, we use Hirota's method.

The calculation results where the space derivative is fractional is shown in Fig.1. Here the dispersion coefficient D = 1.0 and the mean flow velocity v = 0.0, and after 100 time steps. Profiles with a long tail are obtained when the value of alpha small.

We also calculated the time-fractional equation. We found that, by using a small value of gamma, profiles with delay are obtained, which are the result of high adsorption ability.

Finally, fitting with survey data was performed(Fig.2). Rather than the normal advection-dispersion equation, the equation in the present study reproduces the experimental result better. In this example of a column experiment, we found that the order of derivative in space is about 1.5. In the meeting we show comparison with other experimental results, anddiscuss the efficacy of fractional-derivative equations.

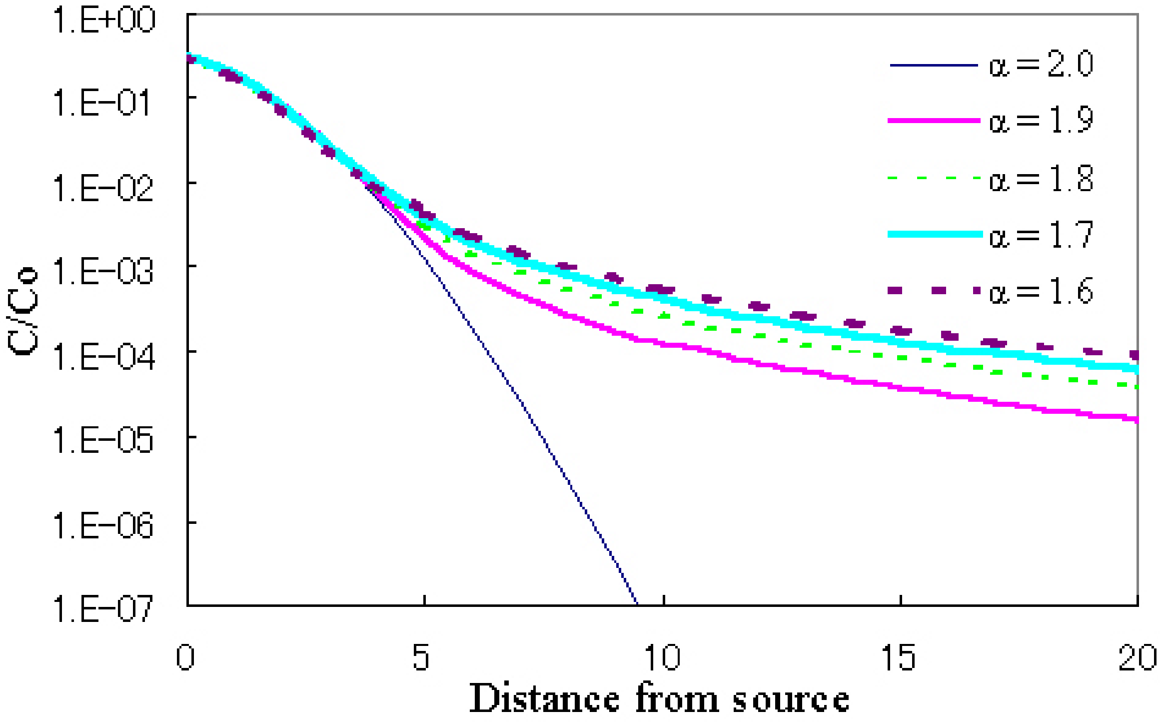

図１　空間でα階の微分をした場合

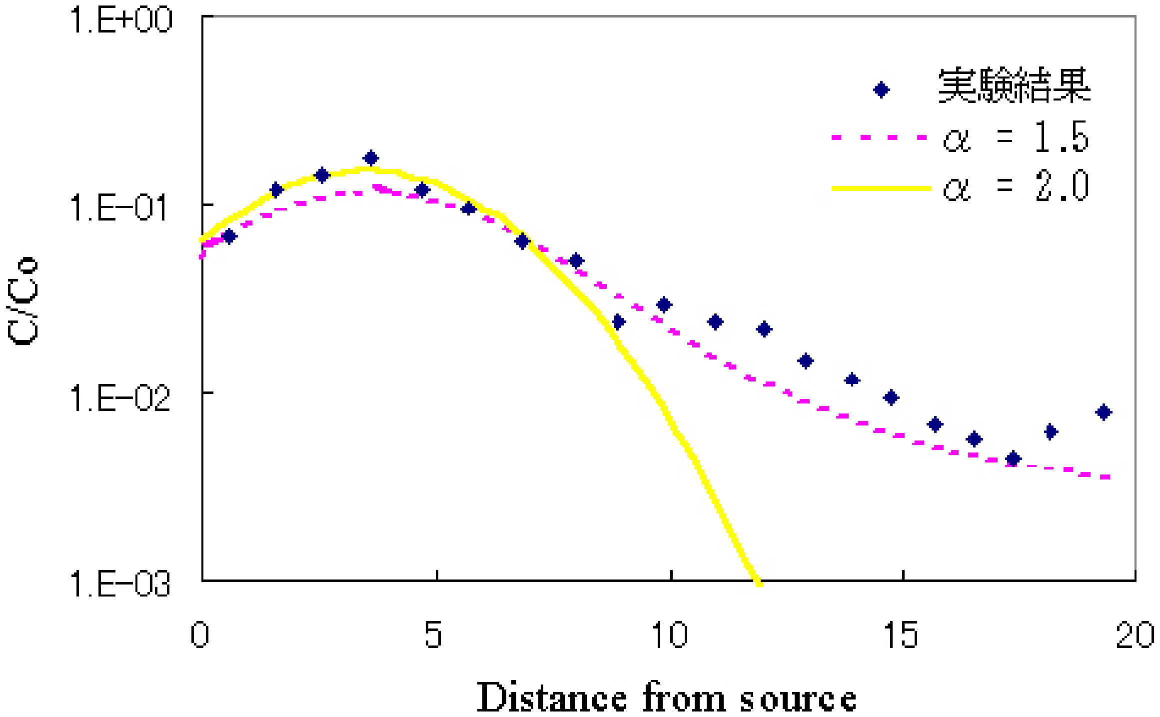

図２　実験データとの比較