

Stretching of material lines and surfaces in turbulence

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Turbulence enhances the mixing and diffusion of passive materials. They have close relations to human life through purification of atmosphere and ocean, combustion efficiency of liquid fuel, and so on. Prediction and control of the diffusion and mixing are of practical importance.

The phenomenon of turbulence mixing is understood as follows. Suppose there are many particles floating in a turbulent flow. Each particle moves randomly and its position changes chaotically. Let us divide the flow domain into two and scatter different materials in each domain. Each material is mingled in a random way. After a while, the two materials are observed with some ratios in a finite domain. This is regarded as a mixing of two materials by turbulence since there was only either one in any finite domain (which does not cross the boundary) at the initial instant of time. If the materials are distributed sufficiently densely or if the fluid is coloured in one domain, this is equivalent to a problem of mixing of fluid itself.

The degree of mixing, or mixing activity, if defined, may be useful for the discussion of the mixing process. However, this is not easy because the above-mentioned mixing rate may depend not only on the position but also on the shape and size of the observed regions. As the first step of characterizing the mixing phenomenon, we study here the deformation and stretching of material lines and surfaces. The importance of the surface may be understood by noting that the combustion rate is proportional to the contact surface of fuel and oxygen.

We examine the deformation and stretching of fluid lines and surfaces in a stationary isotropic turbulence which is simulated by the use of a forced Navier-Stokes equation for an incompressible fluid. The stretching rate of fluid lines is given by the arithmetic mean of infinitesimal line elements which constitute it. For an infinitesimal line element the turbulent velocity may be approximated by linear field, whose gradient is of the order of the inverse of the Kolmogorov time. Both the fluid line and surface increase exponentially in time. The growth rates are independent of the Reynolds number, and are given by $0.17/(\text{Kolmogorov time})$ and $0.28 - 0.30/(\text{Kolmogorov time})$, respectively (Ref. 1).

Each part of a fluid line is stretched locally by the rate represented by the rate-of-strain tensor. Since the turbulence fluctuation has finite correlation time, the stretching rate of each part of fluid lines is regarded as a sequence of random variables with finite correlation time. The total stretching rate of a fluid line experienced over a finite time may be regarded as a random Markovian multiple process composed of many elemental event having finite correlation time (5 Kolmogorov time) of turbulence. Since the logarithm of a Markovian multiple process is a Markovian additive process, the probability density function approaches to a normal distribution after a long time (the central limit theorem). That is, any moments of arbitrary orders of the logarithm of the stretching rate coincides with the moments calculated by the limiting log-normal distribution. It should be noted, however, that the mean value of the stretching rate itself (not its logarithm) is different from those evaluated from this limiting log-normal distribution. This is because the central limit theorem cannot be applied at those part of the probability distribution which contributes the mean value (Ref. 2).

What is the shape of the probability density function of the stretching rate then? It depends on the statistical properties of turbulent fields. In order to estimate it sufficiently accurately, a simulation longer than 10 times of the turbulence correlation time would be necessary. It will be also discussed where and how the fluid lines and surfaces are being stretched more actively in turbulent flows.