A note on interpolation function in grid-based tomography

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Calculation of traveltimes of seismic phases by use of a grid model in heterogeneous Earth has a paradox within, and we propose an interpolation function to avoid this paradox.

Three-dimensional heterogeneity in the interior of the Earth is often represented by grid models or block models. In seismic tomography, geometry of seismic ray paths and traveltimes along them are calculated with these models. In grid models geophysical quantities (e.g., velocity) at grid points are given, and the quantity at arbitrary point is derived by interpolating those at grids. In block models the model space is filled with cellular blocks, to each of which geophysical quantities are given. Grid models are popular in recent studies, since the quantity changes smoothly in space in such models, while the artificial discontinuities occur at boundaries in block models.

Linear interpolation functions are usually used when grid models are adopted in seismic tomography; this is also convenient in linear inversion problems for solving observation equation of traveltime tomography, which usually uses slowness, reciprocal of velocity, as model parameter. It is notable that in usual application of linear interpolation function in such grid models, ray paths and traveltime do not always agree when velocities and slownesses are chosen as model parameters at a grid, respectively. Velocities and slownesses are by definition reciprocal, which are always satisfied on grids. However, at an arbitrary point in model space, the velocity that is given by interpolation of velocities at grids is not always the reciprocal of the slowness that is given by interpolation of slownesses at grids, even with the same function is applied. This is obvious since the arithmetic mean of two different velocities v1 and v2 do not agree with the reciprocal of the mean of two slownesses 1/v1 and 1/v2. This is paradoxical, and the difference is not negligible when the heterogeneity is strong along the ray path, unless grid points are dense enough in space.

There are some ways to avoid this paradox. One of them is to use a geometric mean for the interpolation; however, this induces a non-linear observation equation and is not suited in realistic application of the inverse problem. Hence, we suggest a different interpolation function, which is linear and uses norm estimation in space to measure distances form grid points.

Our interpolation function, which better satisfies the reciprocal relation between the interpolated velocities and slownesses and thus minimizes the differences of ray geometry and traveltimes in two interpolations, is based on maximum norm estimation. Use of this interpolation function equates grid model with block model.