Hugoniot relationship and entoropy condition at magma fragmentation surface

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Fragmentation of magma and generation of a large amount of fine pyroclasts play a significant role in explosive eruptions. We theoretically studied some physical aspects of magma fragmentation on the basis of a 1-dimensional steady conduit flow model. The analyses on the fragmentation surface of steady flows provide us important insights about general physics of magma fragmentation for unsteady flow or instantaneous explosion.

When a volatile-rich magma ascends through a conduit, the volatile component exsolves and bubbles form. As the viscous bubbly magma further ascends though the conduit, the bubbles grow due to overpressure inside the bubbles against viscous resistance. Mitani et al. (this session) has suggested that magma will fragment near the choking condition of the bubbly flow due to (1) large hoop stress due to overpressure inside the bubbles, (2) tensile stress in the liquid bubble wall, or (3) large strain rate of the magma flow. We systematically investigated how Hugoniot relationship and entropy condition depend on these mechanisms of magma fragmentation.

At the fragmentation surface, physical quantities change discontinuously because of burst of bubbles. Before fragmentation (we call State 1 hereafter), the pressure inside bubbles is greater than that of mean pressure (weighted average of gas and liquid pressures), whereas after fragmentation (State 2) the pressure becomes uniform. No relative velocity is assumed in both States 1 and 2. We obtain Hugoniot relationship from mass, momentum and energy conservation across the fragmentation surface. In the present case of steady conduit flows, we know all the physical quantities of State 1 just before fragmentation (see Mitani et al., this session). Therefore, we can determine four unknown variables (velocity, density, mean pressure and internal energy) of State 2 just after fragmentation by four equations (i.e. the above three relationships on conservation and the equation of state for State 2). In practice, we obtain the physical quantities of State 2 as solutions of a quadratic equation. In order that the fragmentation surface exists as a discontinuous surface, the following three conditions must be satisfied; those are (1) the quadratic equation has two real solutions, (2) the mean pressure of State 2 has positive sigh and it is smaller than that of State 1, and (3) when a fluid particle passes across the fragmentation surface its entropy increases.

It is difficult to evaluate the third condition (entropy condition) without knowledge of microscopic phenomena during fragmentation process. We tentatively evaluate the increase of entropy from the macroscopic viewpoints on the basis of the change in the internal energy determined by Hugoniot relationship. When a fluid particle passes across the fragmentation surface, its pressure and internal energy decrease due to expansion. The second law of thermodynamics predicts that the change in the internal energy must be smaller than that accompanied by isoentropic expansion. We, therefore, introduce a new parameter that represents the internal energy change normalized by the internal energy change for isoentropic expansion. From the values of the parameter, it is inferred that the solutions of the quadratic equation are not always consistent with the second law of thermodynamics.

In reality, fragmentation processes involve unavoidable irreversible processes. For example, if gas expands isoentropically just after fragmentation, temperature of the gas becomes lower than surrounding liquid and then the gas and the liquid will thermally equilibrate irreversibly. We evaluate possible changes of internal energy for these processes and discuss the conditions which are consistent with the second law of thermodynamics.