

Fractional governing equation of groundwater flow: a numerical and experimental study

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As the governing equation of the transport phenomenon of contaminants in groundwater, the advection-dispersion equation (ADE) is widely used. However, it has been recognized gradually that ADE does not reproduce some of the experimental results in heterogeneous porous media. Therefore, in the present study, we introduce a new governing equation where the heterogeneity of geologic media is taken into consideration, based on Benson et al.'s (1998) research.

By taking into account of both the heterogeneity of flow and the heterogeneity due to adsorption in the media, advection-dispersion equation modified into the following form, which includes the fractional derivatives.

$$d^{\gamma}C/dt^{\gamma} = Dd^{\alpha}C/dx^{\alpha} - vC/dx$$

The characteristics of the solution of this equation are as follows. While the ordinary advection-dispersion equation has the Gaussian profiles as its solution, this new governing equation gives the profiles described by the Levy distribution. The Levy distribution has a long tail, which is often observed in both laboratory- and field-experiments. We solved numerically the fractional governing equation by the finite difference method, and obtain the solution. We compared the result with a dataset of a column experiment, and found that the derivative (in terms of space) is not 2nd order as in the ordinary ADE, but 1.5.

In order to confirm the fractional ADE, we do experiments in other types of geologic media. The experiment is done in a sand box. The experiment system is built based on Berkowitz and Scher et al. (2000). We first do the experiment, with the medium that does not have artificial heterogeneity, in order to evaluate how the value of alpha relates to the heterogeneity of geologic media. The setup of the experiment is shown in Fig.1. The equipment consists of a 1000mm x 710mm x 120mm container, Marriott tank, a channel switching machine, amplifier, and PC. The sand is packed into a container, and the solution of tracer flows horizontally. The water level of the inlet - and the outlet tank is kept constant by the Marriott tank. The head difference is set at 5cm, and the tracer solution is poured into the sand with the velocity of 1.66×10^{-3} cm/s. The sensor at 15 points embedded in the sand monitors the concentration of the tracer. The electrodes measure the voltage of the solution, and it is converted to the concentration through amplifier, and the result is stored in the PC. At first we use Tohoku silica sand and 3% of NaCl solution, next we use the media with various heterogeneities, with different distribution of the diameter of the media and different degree of the sorption ability. A breakthrough curve is drawn from the experimental result, and we obtain the value of the index of the fractional governing equation.

In conclusions, we show that the type of the governing equation (i.e. the fractional ADE) can describe the transport phenomena in subdiffusive systems.

