

Wave over-reflection applied to the Eady's baroclinic instability

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A simplest model of baroclinic instability was proposed by Eady (1949). In this model, the basic zonal velocity U has a constant vertical shear between the two rigid horizontal boundaries. When we focus on the synoptic scale disturbance, this means that the horizontal temperature gradient is also constant throughout the region. Eady (1949) has got the normal mode solution developing on this basic field. Horizontal scale and the vertical structure of the normal mode well correspond to those of the synoptic scale extratropical cyclones at the developing stage. The deficit of the normal mode approach is that it is difficult to obtain an intuitive image of the mechanism of the instability. Therefore, there were many studies which tried to show the mechanism of the instability. The concept of wave over-reflection proposed by Lindzen et. al. (e. g. Lindzen and Tung, 1978, Lindzen, 1988) is one of such kind of approaches. The heart of wave over-reflection is that the reflection index becomes larger than unity, when the wave tunnels the potential wall which includes the critical layer. Lindzen and his colleagues succeeded in connecting wave over-reflection solution with unstable normal modes for both barotropic and baroclinic basic fields. Takehiro and Hayashi (1992) also applied the concept of wave over-reflection to the shear instability in a shallow water model, and clearly showed that the wave over-reflection is useful to understand shear instability problems. However, there is still no study on the wave over-reflection applied to the Eady problem. One reason is that in the original Eady model, there is no wave region, so it is very difficult to put a wave which could over-reflect. In this study, we succeed to apply the wave over-reflection to the Eady problem, by setting two boundary layers where the Rossby wave exists. We could calculate the wave over-reflection of those Rossby waves. We get the linear unstable mode by using the laser formula. Since we need to assume $ci=0$ when we apply laser formula, the solution has low accuracy when the growth rate is large. However, we can get the dispersion relation with the difference of only factor $\mu \cdot H$, including the short wave cutoff accurately. Applying the laser formula to the solution of the wave over-reflection, we can explain the instability of the Eady's solution as follows. The over-reflected waves perfectly reflected at the tropopause or at the ground surface, and enter the evanescent region again. This process makes the wave growing continuously, and the instability occurs. Through the process, the phase of the incident wave should synchronize with that of the reflected wave, for the efficient growth to happen. This condition is examined by applying the quantum mechanics. In the high wave number case, we cannot synchronize the phase of the waves, and the short wave cutoff has occurred.