Evaluation of numerical errors in the BEM modeling of crack dynamics: the case of self-similar 2-D anti-plane shear cracks

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In the numerical simulation of the dynamic behavior of cracks in an elastic medium, we should always be aware of the presence of numerical errors which are caused by the inevitable discretization of the problem with respect to space and time. In the present study, we take the case of a boundary element method (BEM), developed by Cochard and Madariaga (1994) and enlarged by Tada and Madariaga (2001), to solve, in the time and space domain, the dynamic problem of a planar crack lying in a 2-D, infinite and homogeneous medium. We solve the problem of a self-similar anti-plane shear crack with this BEM and compare the result with a known analytic solution in order to evaluate the size of the numerical errors caused by the discretization.

Suppose that in a 2-D infinite, homogeneous and elastic medium with no dependence on the y-coordinate, an anti-plane shear crack forms at the origin of coordinates at time t=0 under the presence of a yz-component of stress acting at infinite distance, and propagates bilaterally along the x-axis at a constant velocity of 0.8 times the S wave speed. For simplicity, we ignore the effect of friction and assume no stress on the crack surface. This is a sort of the so-called self-similar problem which has an invariable form when one stretches or shrinks the time and space scales simultaneously by an identical magnification factor, and the field of the yx- and yz-components of stress around such a crack was calculated analytically by Kostrov (1964). We evaluated the yz-component numerically with a Gaussian quadrature, as this component is expressible only in the form of an integral on a complex plane.

On the other hand, we calculated the stress field around the above-mentioned self-similar crack by numerical simulations using Cochard and Madariaga's (1994) BEM, which consists in dividing the crack surface into equally spaced discrete elements, discretizing the model time at equal intervals as well, and solving boundary integral equations that link the slip rate profile on the crack with the stress field around it, by applying the 'piecewise-constant' approximation on the slip rate profile so that the latter takes a constant value everywhere inside a given spatio-temporal discrete element. The discretization necessitates that the crack tip, which has to be moving ahead at a constant speed, is modeled as advancing discontinuously by one element size at a time at specific instants.

We calculated, both analytically and numerically, the distribution profiles of the yx- and yz-components of stress and of the maximum shear stress orientation in the medium at two stages where the crack length has grown to 24 and 48 times the discrete element size respectively. Comparison of the results revealed the occurrence of the largest numerical errors in the close vicinity of the crack tips, as well as the emergence of considerably large numerical errors, comparable in size to the amount of stress drop on the crack surface, in the shape of periodically repeated circular arcs over a large area extending from the neighborhood of the crack tips toward both flanks of the crack. The latter phenomenon is thought to be an effect of stress waves originating from the use of intermittently propagating crack tips in the numerical simulations. Little difference was seen in the relative size of numerical errors when we used the two different numbers of discrete elements to cover the entire crack.

These observations reveal that, in the simulation of crack dynamics based on the BEM used in the present study, the use of the discrete crack model with intermittently advancing tips gives rise to stress waves and tends to cause considerable numerical errors, especially in the vicinity of the crack tips. Increasing the number of discrete elements did not lead to large improvements in numerical accuracy, reaffirming the necessity to be aware of the presence of errors whenever one carries out this type of numerical simulations.