

## The Coherency Estimate Method of Short Span Array Wave Data which uses Complex Wavelet Transform, part 2.

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### Introduction

We presented new Coherency Estimate Method on last joint meeting. We applied a wavelet transform method 'Matching pursuit' [Mallet (2000) , Toda (2001)], and expanded it to complex wavelet transform. But there were the difficulty of complex calculation algorithm and necessity to large amount of computer resource for compiling the complex wavelet transform algorithm. We have re-compiling the complex wavelet transform method by applying 'Fast algorithm' [Chui (1997) , Toda (2001)]. We will present this new complex wavelet transform method and some examples of applying complex wavelet transform.

In this study, we have chosen the Meyer wavelet as the function of complex wavelet transform function. Because that function is defined by a simple shape region on frequency-coordinate. You can see the wavelet transformed data's frequency, obviously.

### Method

The Meyer scaling function is 'fsc(t)'. The Meyer wavelet function is 'fwt(t)'. They have two-scale relation.

$$fsc(t) = [\sigma]p(k)fsc(2t-k)$$

$$fwt(t) = [\sigma]q(k)fsc(2t-k)$$

('p' and 'q' indicate coefficient progressions. And  $[\sigma]$  indicates total sum by 'k') ... (1)

The Meyer wavelet has the property of orthonormal base. Below equations explain that.

$$[fwt(2^m t), fwt(2^n t)] = 0 \text{ (if } m \neq n \text{)} \dots (2)$$

$$[fwt(2^m t), fwt(2^n t)] = 1 \text{ (if } m = n \text{)}$$

$$[fwt(2^m t), fsc(2^n t)] = 0 \text{ (if } m \text{ smaller than } n \text{)}$$

('[ , ]' indicates inner product.)

It is known as the unique equation of scaling function decomposition is as follows [Chui (1997)].

$$fsc(2t) = [\sigma](a(k)fsc(t-k) + b(k)fwt(t-k)) \text{ ('a' and 'b' indicate coefficient progressions.)} \dots (3)$$

From upper equations, it is translated the transformed time series data by the scaling function which has an optional scaling value  $2^n$  to the power  $n$  into the wavelet coefficient of  $2^{(n-1)}$  scaling value and the scaling coefficient of  $2^{(n-1)}$  scaling value.

$$xsc(t) = [\sigma](C(k)c(t-k) + D(k)d(t-k)) \dots (4)$$

Definition:  $x(t)$  is time series data.  $c(k)$  is 'scaling coefficient' (=correlation function of scaling functions).  $d(k)$  is 'wavelet coefficient' (=correlation function of scaling function and wavelet function). These definitions are as follows.

$$xsc(t) = [x(t), fsc(2^n t)]$$

$$c(k) = [fsc(2^{(n-1)} t), fsc(2^n t - k)]$$

$$d(k) = [fwt(2^{(n-1)} t), fsc(2^n t - k)] \dots (5)$$

In result, these managements provide the wavelet transformed time series  $D(k)$ .  $C(k)$  is lower frequency component of time series.  $C(k)$  becomes transform's data 'xsc(t)' of lower frequency band.

These are basic method of the Fast algorithm [Chui (1997), Toda(2001)].

We have expanded this method to complex wavelet.

On the third equation of (5),  $fwt(t)$  is expanded to complex wavelet function.

$$dr(k) = [\text{Re}(fwt(2^n t)), fsc(2^n t - k)]$$

$$di(k) = [\text{Im}(fwt(2^n t)), fsc(2^n t - k)] \dots (6)$$

The expanded transform management flow is as follows.

First, regard  $dr(t)$  as  $d(t)$  and use equation 4. And record the extract point of  $dr(t)$  from  $xsc(t)$ . The wavelet transform's real component  $Dr(k)$  is obtained by these management.

Second, extract the imaginary wavelet component from  $xsc(t)$  at the recorded point. There might be 2 imaginary components at a point. In such case, chose one which decreases larger power from  $xsc(t)$ . The wavelet transform's imaginary component  $Di(k)$  is obtained by these management.

Until the  $xsc(t)$ 's RMS level is lower than threshold value, repeat these 2 managements alternately, in large amplitude order.

This flow's management is fast complex wavelet transform method which we made.