General method of computing the wave fields in heterogeneous structure as a frequency wavenumber response characteristics

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Inversion of ACROSS data to the frequency-dependent targets demands a forwards program of computing the wave field with dispersion in a heterogeneous medium. The ordinary Finite Element Time Domain method does not satisfy the requirement. We have happened to find out a supposedly new method of wave field computation as a frequency-wave number response characteristics of a linear dynamic system, which is commonly applied to both elastic and electromagnetic waves. This is the first report on this subject in a series of developing works in the theory and computer code.

Wave equation is interpreted as such that wave field w(t,x) as a function of time t and space x is described as an input and excitation e(t, x) as an input to a linear dynamic system D,

e(t,x) = D(d/dt,d/dx,p1(x),p2(x))w(t,x) (1)

whereas the excitation in wave equation is usually written on the right hand side as an inhomogeneous term of partial differential equation. The D in (1) is an operator containing second derivatives of time and space and a set of two material parameters p1(x)and p2(x). When the material property is discontinuous in space, spatial differentiation in D is executed by using a theory of hyperfunction (or distribution) in deriving the linear system equation, in which the material parameters in D are rewritten in terms of frequency f and wavenumber k, and also by a set of material parameters; slowness, impedance and impedance jump at each discontinuity. When we are to represent the wave field, w(f,k) in frequency and wave number domain, a set of linear system equations in a form of (1) are Fourier-transformed with respect to time and space and the resulted convolution is rearranged to a matrix multiplication;

e(f,k') = ¥sum_k D(f,k',k)w(f,k) (2)

The inverse of (2) leads to a linear dynamic equation, which describes the wave field as an output and the excitation as an input;

w(f,k) = ¥sum_k' R(f,k,k')e(f,k') (3)

where R(f,k,k'), as an inverse matrix of D(f,k,k'), is the 'frequency wavenumber response characteristics'.

The simple approach as above is found to work as evidenced by numerical computation (Nagai et al., companion paper, 2004). It requires too many computations to be practical in general 3D case, and better algorithms are being explored at present.