Efficient numerical analysis of non-planar faults using asymptotic expression of integration kernels in elastodynamic BIEM

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Recently the elastodynamic boundary integral equation method (BIEM) formulated in space-time domain has been applied for the numerical analysis of non-planar faults because its suitability for non-planar problems [Ando et al., 2003, submitted to JGR; Aochi et al., 2000; Kame and Yamashita, 1999]. In the space-time formulation, the stress on the fault surface is represented in term of the slip-rate history as $T(x,t)=-G/2C_sV(x,t)+To+K*V$, where G is the shear modulus, C_s is the S wave speed, V is the slip rate, To is the loading stress, and K is the integration kernel with which slip-rate is convolved over the causality cone in space and time. Evaluation of the convolution integral is the most computationally demanding part of the elastodynamic analysis.

Here we propose an efficient method to reduce computation time in the convolution by using the asymptotic series, much simpler separable expression in time and space, of the functional form of K. For piece-wise constant discretization, the exact representation of K contains two typical functions such as $a(t/r)=[(t/r)^2-1]^{3/2}$ or $b(t/r)=[(t/r)^2-1]^{1/2}$, where t and r are the temporal and spatial distances between a certain source and a receiver, respectively. Each of them has asymptotic series $a(t/r)=(t/r)^3-3/2(t/r)+O[t/r]^{(-4)}$ and $b(t/r)=t/r+O[t/r]^{(-1)}$ within a region $C_pt/r\sim\infty$, which is the far behind from the P wave front (for in-plane problem). Denoting the above asymptotic series as K_{as}, we can find a separable form of K_{as} as $K_{as}(r,t)=fd(r)g(t)+fs(r)const.$ within the region. Note that the original exact kernel is not separable though it is valid over the whole region. By using the separable form of K_{as}, we can reduce the calculation time to an order of N+N from N*N, where N is a total time step and N*N is the calculation time by using the exact kernel. Memory requirement to store the kernel is also reduced to N+N from N*N.

For the special case $g(t)=0$ that appears inside the S wave cone $C_st/r\sim\infty$, we find that $fs(r)$ corresponds to the kernel function, K_{st}, of the elastostatic BIE. Within this special region the double convolution of the slip-rate in space and time reduces to a single convolution of the slip only in space. The similar methodology combining dynamic solution and static solution was originally proposed by Kame [2003, SSJ fall meeting], where the static solution is applied for a condition $t\sim\infty$. However, the convergence of K is uncertain in this condition.

In our new method, the integration kernel is made by combining the following three solutions corresponding to the degree of convergence of K inside the wave cone (see Fig. 1). The exact solution of the kernel K_{es} is used for regions in the vicinity of the P and S wave fronts, $C_pt/r\sim1$ and $C_st/r\sim1$ (K_{es} in Fig. 1), and the convolution is made both in space and in time. For a region that is inside the S wave cone and is far behind from S wave front (K_{st} in Fig. 1), the static kernel K_{st} is used and the convolution is done only in space. For the rest of the cone, i.e the region between the near wave-front regions (K_{as} in Fig. 1), the asymptotic series K_{as} is used. Though the first term $fd(r)g(t)$ is convoluted both in time and in space, its separable form enables us to reduce the computation time from the order of N*N to N+N. The second term $fs(r)const.$ is convolved only in space. The methodology can be readily extended to anti-plane problems with similar asymptotic solutions.

Testing our new method for a self-similar crack problem, we succeeded to constrain the error less than 10% with a set of properly controlled parameters in comparison with a case using the exact kernel. This order of accuracy was also attained in a non-planar crack case. It was finally confirmed that our new method allows significant reduction in the total computation time with accuracy.

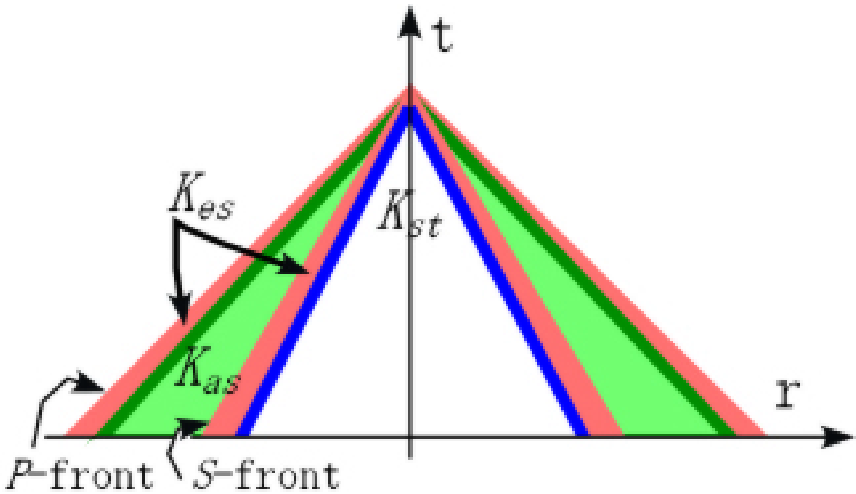

Fig.1 Three types of convolution regions corresponding to kernels, K_{es}, K_{as}, and K_{st}.