General formulation of a method to identify Rayleigh and Love waves in horizontal-component microtremors

Ikuo Cho[1]

[1] G.R.I.

Summary: We have developed a new method to determine phase velocities and arriving directions of Rayleigh and Love waves. The formulation is essentially based on that in Henstridge (1979), which was on the analyses of the vertical component. We extend it for the horizontal components. Our method is general enough to contain the methods of Aki (1957), Okada and Matsushima (1989) or Morikawa (2003) as a special case.

Formulation: Let R(t,r,Q) and T(t,r,Q) be the radial and tangential components at position (r,Q) on the circular array with radius r. We take Fourier expansions of R and T on Q at each time step, so that we obtain the Fourier coefficients that are complex time series. Following is the mathematical expression to this point for R component:

R(t,r,Q)=R_Rayl(t,r,Q)+R_Love(t,r,Q).
where
R_Rayl(t,r,Q)=Integral cos(q-Q)exp{-iwt-irkcos(q-Q)}Z_Rayl(dw,dk,dq), (Z is a integrated spectrum)
R_Love(t,r,Q)=-Integral sin(q-Q)exp{-iwt-irkcos(q-Q)}Z_Love(dw,dk,dq).
The mth Fourier coefficient of Fourier expansion of R(t,r,Q) is expressed as
R_m(t,r) = Integral [A_{m-1}(rk)+A_{m+1}(rk)]exp{-iwt}Z_Rayl_m(dw,dk)
+i Integral [A_{m-1}(rk)-A_{m+1}(rk)]exp{-iwt}Z_Love_m(dw,dk),
where Z_Rayl_m(w,k) is the Fourier coefficient of Fourier expansion of Z_Rayl(w,k,q) on q. A_m(rk)=pi exp{-im pi/2}J_m(rk), where Jm is the mth Bessel function of the first kind.

Likewise, the mth Fourier coefficient T_m(t,r) is expressed as
T_m(t,r) = -i Integral[A_{m-1}(rk)-A_{m+1}(rk)]exp{-iwt}Z_Rayl_m(dw,dk)
+ Integral[A_{m-1}(rk)+A_{m+1}(rk)]exp{-iwt}Z_Love_m(dw,dk).

Next we take power or cross-spectrum of these Fourier coefficients. This enables us to resolve the characteristics of Raylegh and Love waves, or to cancel the phase information. Amplitude information can also be canceled out by taking a spectral ratio of appropriate combination of the Fourier coefficients.

Example: Let the power spectra of T_0(t,r1) and T_0(t,r2) be P(w,r1) and P(w,r2). Then,

P(w,r1)/P(w,r2)=F:E T_0(t+s,r1)T_0'(t,r1)/F: E T_0(t+s,r2)T_0'(t,r2)
....
=J1**2(r1 k_Love(w))/J1**2(r2 k_Love(w)).

F means Fourier transformation, E the expectation, and ' the complex conjugate. In the above derivation, we assume that the wavefield of microtremors is stationary and homogeneous, that the fundamental mode is dominant, and that Rayleigh and Love waves are uncorrelated with each other.

Superiorities of our method:
A. The vertical component is not prerequisite to determine phase velocities of Love and Rayleigh waves.
B. The power ratio of Rayleigh to Love waves is not necessarily prerequisite.
C. Records of the central seismometer are not necessarily prerequisite.
D. We can identify the arriving directions of Raylegh and Love waves.
E. Records from the central seismometer are not necessarily prerequisite also in this case.
F. The effects of array design, including the number of sensors and unevenness in intervals between adjacent sensors, and those of noise existence on the analyses can be theoretically evaluated.