

Evaluation of errors in the numerical simulation of crack dynamics - the case of self-similar 2-D anti-plane shear cracks -

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In basic theoretical studies aimed at elucidating the most characteristic properties of fault dynamics, it is common to numerically simulate the dynamic behavior of faults embedded in an elastic medium under idealized conditions. In doing so, it is unavoidable to discretize the model space with regard to both time and space.

In the present study we consider the problem of a self-similarly evolving anti-plane shear (mode III) crack embedded in a 2-D infinite medium; we numerically calculate, with a boundary integral equation method (BIEM), the fields of stress and displacement velocity created by the crack, and compare the results with known analytic solutions with a view to evaluating the magnitude of numerical errors caused by the discretization.

Assume that slip occurs in the y -direction in an infinite, homogeneous and isotropic medium, in response to an anti-plane shear stress acting at infinity, in a 2-D problem setting that does not depend on the y -coordinate. Let a crack emerge at the origin of coordinates at a given time and propagate bilaterally along the x -axis at a fixed speed. For simplicity, we ignore the effects of friction and assume no stress on the crack plane.

This is a self-similar problem; the y_x - and y_z -components of stress, the y -component of displacement velocity, the orientation of maximum shear stress on the xz -plane and the amount of slip on the crack are calculated both numerically and analytically, and the results are compared.

The analytical solutions for the stress and displacement velocity fields created by a self-similar crack were given by Kostrov (1964) and by Kikuchi (1976); we numerically evaluated, with Gaussian quadrature, the rigorous expression for the y_z -component of stress that is given in the form of an integral on the complex plane.

The numerical solutions for the stress and displacement velocity were calculated with a BIEM, developed by Cochard and Madariaga (1994) and expanded by Tada and Madariaga (2001), that solves the elastodynamic problem of a planar anti-plane shear crack embedded in a 2-D infinite and homogeneous medium. The crack tips, which should be advancing continuously at a fixed speed, were modeled as advancing discontinuously at appropriate time intervals because of the requirement of spatio-temporal discretization.

It was revealed that stress waves, generated by the discontinuous advances of the model crack tips, have considerably large effects, depending on the location on the plane, on the numerical solution for the fields of stress and displacement velocity created by the crack. The effects of numerical errors were particularly serious for the orientation of maximum shear that is calculated, at each sample point, from the magnitudes of individual stress components. This implies that we cannot be too careful of the effects of numerical errors in simulations that involve the orientation of maximum shear, such as in the modeling of spontaneous crack growth. The effects of numerical errors were relatively mild for the amount of slip on the crack except in the vicinity of its tips.

As far as the results of the present study are concerned, the numerical errors did not show characteristic changes in magnitude and spatial distribution profile when we varied the crack propagation speed, but numerical precision was remarkably improved when we refined the grid size and increased the number of discrete elements constituting the model crack. In the latter case, however, the seriously disarranged distribution profile of the orientation of maximum shear showed no recognizable improvement, even with the improved numerical precision for individual stress components, because of the strong local effects of the stress waves.

Insights obtained in the present study are expected to provide implications to a number of related research subjects, since numerical methods of the same category are widely in use to deal with the cases of 2-D in-plane, 3-D and non-planar crack problems.