Finite scale effect on percolation threshold for elongated bubbles

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The permeability of vesiculating magma is an important factor of degassing process from volcanic conduits. Recent experimental studies using rock samples (typically several to twenty cm in length) and numerical studies for small porous medium (less than 100 times bubble size) suggest that deformation of bubbles largely affects the permeability-porosity relationships in volcanic rocks (Saar and Manga, 1999; Blower, 2001); the permeability along the major axis of elongated bubbles increases with the degree of elongation, and the percolation threshold decreases. On the other hand, it is known that the percolation threshold and hence permeability strongly depend on the scale of the porous medium. Therefore, it is still an open question whether the above results for small porous media are applicable to volcanic conduits of more than 10 m. We analytically study the scale effect on permeability of porous magma containing elongated bubbles. Generally speaking, permeability is sensitive to porosity change near the percolation threshold. Thus we focus on the dependence of the percolation threshold for elongated bubbles in a finite porous medium on the scale of the medium in this study.

On the basis of a recent percolation theory, we derive a formula for percolation threshold (Pc) for elongated bubbles in a finite porous medium along the major axis as;

$$Pc(L,R) - Pc(L,1) = - A L^{(-1/U)} (\ln R)^{(1/B)},$$

where L is the scale parameter defined as the ratio of the size of the porous medium to the half length of the bubble major axis, R is the aspect ratio of the bubble, and U, A and B are coefficients whose values should be specified numerically. According to the percolation theory, U is known as a universal constant depending only on the space dimension and its value is 0.9 in 3D. We numerically determined the values of A and B as 1.4 and 1.5, respectively. The left hand side of the formula is the difference between the percolation threshold for elongated bubbles (aspect ratio is R) and that for spherical bubbles (aspect ratio is 1) in a finite medium (scale is L), and it represents the degree of anisotropy of the percolation threshold due to bubble elongation. The result shows that the degree of the anisotropy increases as the bubble aspect ratio increases; however, such an effect of bubble elongation diminishes as the scale increases. It is suggested that the effect of bubble elongation on the percolation threshold is unlikely to be detected when the size of the porous medium is 10^3 times larger than bubble size.