

## Derivation of scale dependency of fracture energy from a hierarchical self-similar geometry of fault zones

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Yamada et al. (2005, JGR, v. 110, B01305) analyzed micro-earthquakes, magnitude from 0.8 to 1.4, observed in the deep gold mines of South Africa, and confirmed that even for such small earthquakes fracture energy scales linearly with the size of fractures. On the other hand, Otsuki and Dilov (2005, JGR, v.110, B03303) that fault zone geometry is self-similar in which fault segments and jogs are nested hierarchically, and they derived two empirical laws; Gutenberg-Richter law and seismic moment proportional to the seismic nucleation size to the power of 3. Here I show that the size dependency of fracture energy also can be derived from the fractal fault zone geometry. Based on the fact that fault segments and jogs are hierarchically nested, my fundamental concept is that seismic slips start at a jog of a given hierarchical rank  $i+j$ , destroy all of the segment+jog structures of lower ranks than  $i$ , and eventually stop at jogs of the hierarchical rank  $i$ , and that the mean fracture energy  $G$  will be proportional to the volume density of all jogs destroyed to the power of  $b$  divided by the area fractured, namely  $G = c(\text{length} \times \text{width} \times \text{thickness of jogs})^b / \text{fractured area}$ .

Otsuki and Dilov (2005) found the relationships of segment length  $LS(k)$ , total number of segments  $NS(k)$ , jog length  $LJ(k)$ , step-over distance of a jog  $TJ(k)$  and total number of jogs  $NJ(k)$  for a given hierarchical rank to the total length of a fault zone  $L_0$  for the experimental fault zones. Here  $L_0$  must be change to  $LS(i)$ , and the relations above must be rewritten to the relations of  $LS(i)$  to  $LS(i+j)$ ,  $NS(i+j)$ ,  $LJ(i+j)$ ,  $TJ(i+j)$ , and  $NJ(i+j)$  with  $LS(i)$ . Otsuki and Dilov (2005) based on the 2-D observation, parallel to the slip vector and perpendicular to fault planes. Here we extend it to a 3-D problem, assuming that segment width  $WS(k)$  also is self-similar (equal to  $r \times LS(k)$ ), and that jog width  $WJ(k)$  is equal to the segment width  $WS(k)$ . Finally the equations below are derived.

- (1)  $LS(i+j) = CSL^j * LS(i)$
- (2)  $NS(i+j) = CSL^{-2j}$
- (3)  $LJ(i+j) = CJL^j * LS$
- (4)  $TJ(i+j) = CJT * CSL^{a(j-1)} * LS(i)^a$
- (5)  $NJ(i+j) = (1-CSL) * CSL^{-j}$
- (6)  $WJ(i+j) = r * CSL^j * LS(i)$

There are  $NJ(i+j)$  jogs of rank  $(i+j)$ . Following the assumption above, the energy necessary for one jog to be fractured is proportional not only to  $LJ(i+j) * WJ(i+j)$  but also to  $TJ(i+j)^b$ . Therefore, the energy  $E(i+j)$  necessary for all jogs of rank  $(i+j)$  to be fractured is,

$$(7) E(i+j) = c * LJ(i+j) * WJ(i+j) * TJ(i+j)^b * NJ(i+j)$$

We assumed that only one segment of rank  $i$  is fractured and the jogs of rank  $i$  stop it. Jogs of rank  $i+j$  from  $j=0$  to infinite are nested hierarchically in a segment of rank  $i$ . Therefore, the sum of these  $E(i+j)$  divided by the area of a segment of rank  $i$ ,  $rLS(i)^2$ , is equivalent to fracture energy  $G$ . Substituting eqs. (3) to (6) to eq(7), we have,

$$(8) G(i) = c * CJT * (1-CSL) * [\text{Sum: } CSL^{ab(j-1)} * CJL^j] * LS(i)^{ab}$$

Since  $CSL (=0.343)$ ,  $CJL (=0.0935)$ ,  $CJT (=0.0456)$  and  $a (=0.642)$  are constants, and  $b$  also was assumed as a constant, both the term inside [ ] and the prefixed terms of the right side of eq. (8) are constant for  $i$ . Therefore, we conclude that fracture energy  $G(i)$  increases proportionally to the length of a fracture  $LS(i)$  to the power of  $ab$ .

The derivation above is based on the assumption that the local fracture energy at jogs is proportional to the step-over distance  $TJ(i+j)$  to the power of  $b$ . Even if this assumption is acceptable, the value of  $b$  is unknown yet. If fracture energy scales linearly with the size of fracture,  $b$  is expected to be about 1.5, because  $a$  was estimated at 0.642 by Otsuki and Dilov (2005). Is it acceptable or not?