

Equivalence of the Born- and Foldy-approximated solutions for scattering attenuation due to weak discrete inhomogeneities

Jun Kawahara[1]

[1] Faculty of Science, Ibaraki Univ.

There are two major theoretical approaches for predicting the seismic scattering attenuation due to the lithospheric random inhomogeneity. In one approach, inhomogeneous media are modeled by materials with weakly and randomly perturbed elastic wave velocity and density (termed random media) and the single-scattered energy due to the perturbation is evaluated on the basis of the Born approximation (Chernov 1960). Note that the perturbation is described using the autocorrelation function. Chernov's theory was introduced into seismology by Kei Aki's group, and then has been developed in a unique manner, characterized by the introduction of traveltimes correction (Aki & Richards 1980; Sato & Fehler 1998). In the other, the mean wave formalism is adopted for treating the coherent wave propagation and the multiple scattering process within a disperse system of discrete inhomogeneities, such as suspended particles, bubbles and inclusions. The first-order approximated solution for this problem was first obtained by Foldy (1945). Foldy's theory was applied to scattering due to crustal cracks by Kikuchi (1981), and its validity range has been discussed (Kawahara, 2001).

Note that the scattering attenuation in random media can be also evaluated using the mean wave formalism (Karak & Keller 1964), though the results are proven to agree with the (traveltimes-uncorrected) Born-approximated solutions (Wu 1982). On the contrary, one can also formally describe a disperse system in terms of the autocorrelation function and thus estimate scattering attenuation using the Born-approximation. If the distribution were very sparse as well as the material contrasts between the inhomogeneities and the surrounding matrix were sufficiently low, such an approach would give results equivalent to the Foldy-approximated ones, the virtually correct attenuation. To the author's knowledge, however, seems no example to demonstrate this inference. Its proof is the purpose of this study, though restricted to some special simple model geometry. We do not consider the traveltimes correction here.

We treat here SH wave scattering attenuation due to randomly distributed 2-D circular inclusions with the same size. For the spatially averaged S wave velocity of the composite medium to be equal to the matrix S wave velocity c_0 , we assume that half of the inclusions have the velocity c_0+dc and the rest do c_0-dc , with dc being a small positive value. We do not consider the density perturbation for the moment. We also assume the distribution to be sufficiently sparse so as to make the Foldy approximation reliable. The scattering amplitude of a circular inclusion has been analytically given by Pao & Mow (1973), and hence the Foldy-approximated solution is calculated straightforward (Kawahara, 2001). The Born-approximated counterpart can be estimated following the manner of Aki & Richards (1980); the exact autocorrelation function for the present case is given by Stoyan et al. (1995). The calculation reveals that the scattering $1/Q$ values based on both approximations are highly consistent in the limit of vanishing dc/c_0 . Generally, they are approximately equal within the Rayleigh-Gans scattering regime, namely, ka less than c_0/dc , in which ka is the wavenumber normalized by the inclusion radius. For ka beyond c_0/dc , the Foldy-approximated $1/Q$ reaches a peak and then decreases, whereas the Born-approximated one incorrectly keeps rising because it does not take account of the discreteness of inclusions, or the phase spectra of the perturbation. These conclusions are not changed by additional weak density perturbation.

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