Theoretical derivation of plane-wave envelopes in 3-D random elastic media characterized by Gaussian autocorrelation function

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High-frequency seismic waves propagating through the lithosphere are scattered and diffracted by its random inhomogeneity. Even though seismic wave is impulsive at the time of radiation from a source, its apparent duration time is broadened with travel distance increasing. Envelope broadening has been studied based on the Markov approximation for the parabolic wave equation for scalar waves and a stochastic extension of the phase screen method. Recently, we succeeded in extending the Markov approximation to vector waves in 2-D random elastic media, and confirmed the validity of this method by using finite difference simulations.

Here, we extend the formulation to vector wave propagatinon through 3-D random elastic media. Let imagine an ensemble of random elastic media characterized by a Gaussian autocorrelation function. We statitially study the propagation of a plane wavelet with wavelength shorter than the correlation distance through the media, when we may neglect conversion scattering between P and S waves. We define the two-frequency mutual coherence function (TFMCF) of potential field, where the MS envelope of each vector component is given by using its Fourier transform. We can analytically solve the master differential equation for TFMCF and analytically perform its Fourier transform. Finally we obtian the analytical representation of the MS envelope of each component by using the elliptic Theta function. For the case of plane P-wavelet incidence, there is a peak delay from the onset and a smooth decay after the peak in the logitudinal component after a long travel distance, where the characteristic time is proportional to a product of the square of fractional fluctuation of velocity and that of travel distance over a product of the correlation distance and the average P-wave velocity. The ratio of the maximum peak of the transverse MS envelope to that of the longitudinal MS envelope is given by the ratio of a product of the square of fractional fluctuation of P-wave velocity and the travel distance to the correlation distance. The transverse-component amplitude is a good measure of randomness, since the time integral of its MS envelope linearly increases with travel distance increasing. For the case of plane S-wavelet incidence, the same procedure is applicable to a component of the vector potential. When the fractional fluctuation of S-wave velocity is the same as that of P-wave velocity, the envelope broadening is larger for S-waves than P-waves since the characteristic time is proportional to the reciprocal of wave velocity. This representation is useful for the analysis of teleseismic wave envelopes for the study of the lithospheric inhomogeneity.