# Envelope broadening and travel distance dependence of peak delay time of sphericalwaves in non-uniform random media 

\# Tsutomu Takahashi[1]; Haruo Sato[1]; Takeshi Nishimura[1]

[1] Geophysics, Science, Tohoku University

Seismic waves of microearthquakes are impulsive when they are radiated from their sources, however, they collapse with travel distance increasing due to random inhomogeneities in the lithosphere. It is very useful to analyze their envelopes neglecting phases. Wave envelopes around their maximum amplitudes are well described by the Markov approximation for the parabolic wave equation, which is a stochastic extension of the phase screen method. This approximation is able to uniquely predict wave envelopes from the statistical parameters characterizing random inhomogeneities. However, all the established theoretical models were valid only for spatially uniform random media, which is far from realistic cases. There is alternative simulation method called the stochastic ray-path method developed by Williamson (1972), which describes the ray bending caused by random inhomogeneity of velocity as a stochastic process. This approach is useful to consider spatially non-uniform random media.

In this study, we develop a Monte Carlo simulation to synthesize spherical wave envelopes based on stochastic ray-path method in random media characterized by a von Karman type power spectral density function (PSDF). By using this method, we examine the characteristics of seismic envelopes propagating in spatially non-uniform random media, where the PSDF of random inhomogeneity varies along the seismic ray-path. If the correlation length of inhomogeneous media is longer than wavelength and the root mean square of velocity fractional fluctuation is small, forward scattering becomes dominant and waves are governed by a parabolic wave equation. Taking the ensemble average of wave correlation, we obtain the first-order differential equation of mutual coherence function. For a spherical source radiation case, we divide the whole space into many spherical shells with small thickness having a center at the source. Ray bending caused by velocity inhomogeneity in each layer is described as a stochastic process, where the probability density function controlling the scattering angle of seismic ray is derived from PSDF of random inhomogeneities. By using Monte Carlo approach, the mean square envelopes can be evaluated from the histogram of particle arrival times at a given travel distance for isotropic shots of many particles from the source.

For the case of spatially uniform random inhomogeneities, we confirmed that simulated envelopes show good agreement with envelopes theoretically derived by using the two-frequency mutual coherence function. Then, we synthesize envelopes in random media in which inhomogeneities varies along the seismic ray-path. To characterize the envelope broadening, we define the peak delay time which is measured from the direct wave onset to the maximum amplitude arrival. We measure how the peak delay time varies with travel distance increasing. The peak delay time increases as a power of travel distance, and its power depends on the spectral decay parameter of the PSDF in short-wavelength components in the case of uniform randomness, however, the power of travel distance depends on many parameters characterizing PSDF in the case of non-uniform randomness. Here, we propose a simple rule which enables us to estimate the peak delay time for any case that randomness varies along seismic ray-path. We divide the whole medium from the source to the receiver into many segments, where the randomness in each segment can be considered to be uniform. After traveling the first segment, we replace the peak delay time as a function of an equivalent travel distance at the boundary as if the ray traveled in a random medium having the same characteristics as the medium in the second segment. Repeating this replacement at all the segment boundaries, we obtain the peak delay time at the receiver. This rule is useful for the inversion of peak delay times for the spatial distribution of statistical parameters characterizing randomness.

