

## Equivalence theorem of linear viscoelasticity: Completely relaxed viscoelastic solution and the associated elastic solution

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We can regard the earth as an elastic body, as long as our concerns are limited to short-term crustal deformation, such as coseismic deformation. To long-term crustal deformation such as interseismic deformation and postglacial rebound, however, we must consider the viscoelastic property of the asthenosphere even for the first approximation.

Viscoelastic response due to some force, such as a surface load and a dislocation source, can generally be obtained on the basis of the correspondence principle of linear viscoelasticity (Lee, 1955; Radok, 1957). According to the correspondence principle, we can directly obtain the viscoelastic solution from the associated elastic solution on the Laplace plane. In order to obtain the viscoelastic solution on the time domain, however, we have to operate the inverse Laplace transform on the viscoelastic solution on the Laplace plane. The inverse Laplace transform is usually very complicated, and it is often impossible to perform analytically.

The viscoelastic deformation due to some force tend to a certain steady state with the progress of viscoelastic stress relaxation. In some cases, it is enough to obtain the completely relaxed solution only. In general, we can obtain the completely relaxed viscoelastic solution on the time domain, by applying the limiting value theorem of the Laplace transform. The limiting value theorem relates the completely relaxed viscoelastic solution on the time domain to the viscoelastic solution on the Laplace plane.

In this study, using both the correspondence principle of linear viscoelasticity and the limiting value theorem of the Laplace transform, we give mathematical proof that the completely relaxed viscoelastic solution on the time domain can directly be obtained from the associated elastic solution. We named this theoretical relation 'equivalence theorem'. Based on the equivalence theorem, we can deduce various results.

For example, for a body with (an) elastic media (medium) and a Maxwell viscoelastic medium (a typical model for the elastic lithosphere and the viscoelastic asthenosphere), the completely relaxed viscoelastic solution exactly coincides with the associated elastic solution with the corresponding elastic layer having zero rigidity. This means that the Maxwell viscoelastic medium cannot support any deviatoric stress after complete relaxation. In addition, we can show that the completely relaxed solution does not depend on the value of viscosity, which just controls the speed of viscous relaxation.

For the case with (an) elastic media (medium) and more than one Maxwell viscoelastic medium (one of the typical models for the elastic upper crust, viscoelastic lower crust and the viscoelastic asthenosphere), the completely relaxed viscoelastic solution also coincides exactly with the associated elastic solution with the corresponding elastic layers having zero rigidity. However, we need to pay special attention to taking the limit of rigidity to be zero, because the relaxation speed in each layer depends on its viscosity. Actually, we must take the limit of rigidity to be zero, keeping the ratio of rigidity for each layer to be equal to the ratio of viscosity for each layer. Thus, the completely relaxed solution in this case depends on the viscosity.

We can apply the correspondence principle to general elastic linear-viscoelastic composite media. Owing to the correspondence principle, we can obtain the completely relaxed solution, without passing through complicated viscoelastic calculation. We will show some results of computation in the presentation. Incidentally, this study is a generalization of the paper entitled 'Equivalence theorem in a Maxwell viscoelastic problem', given at the fall meeting (1998) of the Seismological Society of Japan.