

A parameter space for stress tensor inversion

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Stress tensor inversion techniques using fault-slip data aim to determine principal stress orientations and a stress ratio, or equivalently a reduced stress tensor. The stress ratio can be defined as $\Phi = (\sigma_2 - \sigma_1) / (\sigma_3 - \sigma_1)$. The expression by principal axes and stress ratio is not suited to enumerate candidates of stress solution, since the effect of change in orientation depends on the stress ratio. To address this problem, Sato and Yamaji (2006a) proposed a five-dimensional (5-D) parameter space in which points on the unit sphere have one-to-one correspondence to reduced stress tensors and the distances between points are equal to a measure of difference between stress states called 'stress difference' (Orife and Lisle, 2003). The stress difference appears to be a suitable measure for our problem because it can predict the difference in slip direction of a randomly oriented fault (Yamaji and Sato, this meeting).

The new formulation brings us the following benefits.

(1) A set of uniformly distributed points on the 5-D unit sphere gives an unbiased candidates of stress solution (Sato and Yamaji, 2006b). This set of points enhances the efficiency of grid search. Furthermore, it was found that the set includes larger number of triaxial stresses ($\Phi \sim 0.5$) than axial ones ($\Phi = 0$ or 1). This implies that the 3-D stresses have larger variation in orientation than 2-D (axial-symmetric) stresses.

(2) The stress tensor inversion problem is interpreted into a concise geometrical problem. The reduced stress tensors which are concordant with a fault-slip datum are found to be located on a great semicircle on the 5-D unit sphere (modified after Fry, 1999, 2001). This fact leads us to a straightforward method of inversion using Hough transform (Yamaji et al., 2006). The great semicircles specified by faults are superposed to compose a fitness distribution on the unit sphere, and the point which maximizes the fitness represents the optimal stress. If there are multiple peaks in the fitness distribution, we obtain the multiple stress states recorded in a 'heterogeneous' fault-slip dataset.

(3) Imperfect fault-slip data can contribute to constrain the unknown stress. On outcrops, we recognize ancient faults by offsets of key beds. If the faulted blocks had adhered, the orientation of slippage is not observed as scratches on the fault surface, while we have the information on the shear sense, i.e., whether the fault is normal or reverse one. Conversely, if there is no marker of offset, the shear sense can not be judged. The loose constraints of such imperfect data on stress are also specified as large admissible regions of reduced stress tensors in the parameter space. Then the superposition of admissible regions of perfect and imperfect data gives the total fitness distribution. Seismic focal mechanisms can also be regarded as imperfect data, since two nodal planes can not be distinguished to be fault planes. The admissible regions from two nodal planes are combined and weighted as one datum in the superposition.

(4) Applications of statistical techniques are facilitated. As an example, we introduce a method to estimate the number of stress states recorded in a heterogeneous fault-slip dataset (Sato, this meeting). Firstly, EM algorithm fits a model of mixed 5-D von Mises-Fisher (vMF) distribution to the fitness distribution obtained by Hough transform. Secondly, the optimal number of mixed distributions is determined by MDL information criterion. The stabilities of determined stress solutions are evaluated by the concentration parameters of vMF distributions. Note that the stability is measured by stress difference.

The present parameter space sheds light on the numerical processing in stress tensor inversion, and it may be useful to assess the inversion techniques by expressing them in 'visualized' forms.