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Regularity Problem of Solutions to the Incompressible Fluid Equations

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Despite their long history, satisfactory existence theorems for the 3D Euler and Navier-Stokes equations have not been obtained so far. We will start off by briefing on the physical rationale for considering such basic issues.

We then review why conventional mathematical techniques fail to settle the problems. (1) For the inviscid 3D Burgers equations, a method of characteristics can be employed to show appearance of singularity, i.e. crossing of characteristics. But for the Euler equations, because of nonlocality associated with pressure term, the characteristics are bent and we do not know whether they cross in finite time or not. (2) For the (viscous) 3D Burgers equations, a maximum principle works and global regularity is guaranteed. However, for the 3D Navier-Stokes equations, again by the pressure term such a principle is invalidated.

If we try to bound the growth of enstrophy (L^2-norm of vorticity) by using calculus inequalities, we end up with an inequality which becomes meaningless in finite time (nonlinearity eventually dominates over viscosity). The unfortunate situation is the same for 3D Burgers equations, whose regularity is established only by the maximum principle. Because the inequality is optimal, in the sense that it is dimensionally consistent, this failure is serious. Essentially different techniques will be required to control the enstrophy properly.

We then turn to modern results. For the Euler equations, we review the celebrated result by Beale-Kato-Majda(1984), which stresses the importance of vorticity maximum in controling the regularity of flow fields. Then we will review advances developed since then. Specifically, we pay attention to the results which say something about spatial structure of a flow under consideration. This includes a result by Constantin et al. who showed that if the vorticity direction is well-defined in the high-vorticity regions then the flow remain regular. It also includes an update of sup-norm in the Beale-Kato-Majda result by BMO (Bounded Mean Oscillation)-norm, which is typically represented by a logarithmic function. Thus, now we know that a logarithmic singularity in vorticity is ruled out.

In connection with turbulence theory we also address so-called Onsager conjecture for conservation of energy in weak form. It means the following: for a weak solution of the 3D Euler equations to conserve total energy, it is necessary and sufficient that velocity is more regular than the Kolmogorov scaling. The sufficiency has been established. The necessity remains an insuperably difficult problem, because we do not know how to establish existence of weak solutions.

Next, for the Navier-Stokes equations, we review some partial results for their regularity, including absence of self-similar blow-up solutions. Back in 1970, Rosen deduced a condition to rule out self-similar blowup (demanding some smoothness). Necas et al.(1996) gave a more satisfactory result by showing that if velocity is in L^3, i.e. cubic integrable, then self-similar blowup is precluded.

Finally we may briefly address the relevant problem of magnetohydrodynamics.