WKB analysis of Axisymmetric Magneto-Rotational Instability in a Thin Accretion Disk

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Abstract : The temporal behavior of axisymmetric magneto-rotational instability in a thin accretion disk is analyzed via the Wentzel-Kramars-Brillouin (WKB) method. The height of the thin disk is used as a small parameter. It is found that the oscillation of the envelope of the mode accelerates with time because of the density distribution in the direction of the disk height.

Magneto-rotational instability (MRI) [1,2] in accretion disks [3] has attracted much attention in astrophysics research since Balbus and Hawley pointed it out as a candidate for explaining the “anomalous” angular momentum transport in accretion disks [4]. In Ref.[4], the MRI was studied by local analysis which assumes a sinusoidal wave in the radial and height directions of the accretion disk. The global mode was discussed in, for example, Ref.[5]; however, they also adopted a sinusoidal wave in one of the radial and height directions. It is also noted that most of the MRI studies, except for the nonlinear numerical studies, are based on the eigenvalue approach. In the present study, we analyze the temporal behavior of the axisymmetric MRI in a thin accretion disk as an initial-value problem. The Wentzel-Kramars-Brillouin (WKB) method is applied and the sinusoidal wave is assumed in neither the radial nor height directions. The height of the thin disk is utilized as a small parameter in the WKB analysis.

Here we adopt the ideal magnetohydrodynamics (MHD) model. In the equilibrium, we assume a constant magnetic field in the vertical direction and a velocity field in purely azimuthal direction. The temperature of the plasma is also assumed constant. If the rotation velocity is much larger than the sound speed, the disk has a thin structure. The self-gravity is neglected.

In the stability analysis, we focus on a mode with three scale length; (1) a large scale in the radial direction, (2) an intermediate scale in the vertical as well as the radial direction, and (3) a short scale in the vertical direction. The first one is comparable with the equilibrium scale length in the radial direction, the second one with the typical height of the accretion disk, and the last one is much shorter than them. In order to express such a mode structure, we introduce the WKB method for the plasma displacement. The typical height of the disk is used as an expansion parameter. Then, we obtain, in the lowest order, the local dispersion relation which is the same as the conventional one. In the next order, we obtain an evolution equation for the envelope. We can integrate this equation in time to obtain the time evolution of the envelope. We have found that the oscillation of the envelope becomes increasingly faster in time because of the density distribution (or the variation of the Alfven velocity) in the vertical direction.

In conclusion, we have succeeded in capturing the transient phenomena of axisymmetric MRI for the first time; this has not been captured using the eigenvalue approach in long cylinder geometry. Although we have used such a simple equilibrium magnetic field, it may be enough to point out the significant importance of the global treatment of MRI as well as the initial-value approach.

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