An analysis of unsteady one-dimensional conduit model for lava dome eruptions

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During lava dome eruptions, periodic ground deformations with periods of a few ten hours (e.g., the Unzen 1991-1995 eruption), and periodic changes of magma effusion rate with periods of a few ten years (e.g., the Santiaguito 1992-2000 eruption) are observed. From these observations, we can infer the changes of pressure in magma chambers (P) and those of magma flow rate through conduits (Q). So far many models have been proposed in order to explain the changes of P and Q. These models are classified into two types: (1) to solve spatial distributions of physical quantities along conduits at steady state and to investigate nature of the relationship between P and Q at the steady state ('steady P-Q curve') (e.g., Woods and Koyaguchi [1994]); (2) to carry out the linear stability analyses for 2-dimensional (2-D) dynamical systems where the variables (e.g., P and Q) are defined as a spatially averaged quantities in conduits, and to investigate the critical conditions for steady flow to be unstable so that oscillation occurs ('stability condition') (Whitehead and Helfrich [1991], Ida [1996], Wylie et al.[1999] and Maeda [2000]). These approaches have disadvantages as follows. In the former approach, because it is based on the equations for the steady state, the stability conditions cannot be determined. In the latter approach, because the variables used in the models are spatially averaged quantities, it is difficult to understand physical meanings of the estimated stability conditions. In order to resolve these disadvantages, we attempt to obtain a universal relationship between steady P-Q curve and stability conditions. Here, we focus on such relationships in 2-D dynamical systems.

Since previous models of 2-D dynamical systems are designed to analyze periodic changes due to different physical factors, the variables used in these models are not always consistent, which makes it difficult to compare the results of the stability analyses between the different models. Therefore, we reformulated them using common variables: P and Q. As a result, we found that these models have two common features: (1) the time evolution of P is governed by the balance between the magma flow rate through the conduit Q and the magma supply rate into the magma chamber Qin, and (2) the relationship between P and Q is represented by P=VQ (i.e., Poiseuille flow) where V is the average viscosity. We also found that the differences between the models are explained by the differences in the expression of V as a function of P, Q and Qin. This formulation using P, Q and V allows us to carry out the stability analyses for all the previous models on a common basis.

In general, the stability conditions of 2-D dynamical systems for P and Q can be investigated using the curve of dP/dt=0 ('P-nullcline') and that of dQ/dt=0 ('Q-nullcline') on the basis of the linear stability theory. Since the P nullcline is Q=Qin (i.e., a vertical line in the Q-P space) in the above formulation, the stability condition is given by the condition where Q nullcline has a negative slope at the fixed point (i.e., the intersection of P and Q nullclines). On the other hand, a steady P-Q curve of a two-dimensional dynamical system can be obtained from the set of the fixed points for variable Qin, and hence, the steady P-Q curve is not identical to the Q nullcline. Therefore, generally speaking, we cannot determine the stability condition from the steady P-Q curve. We found a simple formula to express the relationship between the slope of Q nullcline at the fixed point and that of steady P-Q curve for the case where V is independent of Qin and depends only on P and Q. In this case we can determine the stability condition from the slope of steady P-Q curve. We also found a universal relationship between the stability condition and the periods at the bifurcation points for this particular case.