

Application of Algebraic Structure to Spatial Patterns of Microfracturing in Rock

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Geometric analysis of spatial distributions of microfracturing in rock (acoustic emissions) has been applied usefully to other similar fracture events in nature such as earthquakes (e.g., Hirata et al., 1987; Lockner et al., 1991; Lei et al., 2004). In this work, we reconsider the spatial patterns of microfracturing in rock from a viewpoint of algebra. The geometric patterns of spatial distribution of microfracturing in rock are interpreted by using the algebraic structures, i.e., lattices defined as ordered set and binary operations join and meet. The spatial distribution of microfracturing in rock has been measured quantitatively based on the symmetry properties of the two-dimensional discrete Walsh functions (Nanjo et al., 2000). For algebraic interpretations, the two-dimensional discrete Walsh functions are transformed into concept lattices which stem from the so-called formal concept analysis - a pair of applied mathematics that brings mathematical methods into the field of data analysis and knowledge processing (Schwarzweiler, 1998; see also Ganter and Wille, 1996). In the transformation, formal concept analysis can be applied to the first 16 patterns (If these patterns are divided into four sorts, they are called the symmetry properties) of the two-dimensional discrete Walsh functions that consist of 4x4 rectangular cells respectively, since 16 patterns can be regarded as cross table. It is found that two sorts of concept lattices, we named lattice diamond (LD) and lattice pole (LP) respectively, replace the four sorts of symmetry properties. Moreover, concept lattices show that the fracture process (creep before ultimate whole fracture) can be interpreted algebraically as a transition from LD to LP. We proved the mathematical consistency of the transition in terms of the sheaf: the basic tool for expressing relationships between small regions and large regions of a space, in other words, an operation to group parts. Thus it is concluded that concept lattice, one of algebraic structures, is useful for the study of spatial patterns of fracture events including microfracturing.