Numerical calculation of wave fields with frequency-wavenumber response

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A new theory to calculate wave fields has been developed in order to provide a support for sinusoidal approach in geophysical exploration and also a potential method of analyzing the very accurate data acquired by ACROSS (Kumazawa et al., 2004). In the use of the theory, wave equation described by differential equation is rewritten to a linear system model expressing the external excitation as output and the wave field as input, then the linear equation is converted to an inverse system model expressing the external excitation as input and the wave field as output in the frequency and wavenumber domains. This inverse system model is described by FWR, the frequency and wavenumber response. Multiplying the excitation by the FWR, we obtain the wave field as a dispersion relation.

The numerical examples are presented to validate the new theory using the simplest case, a finite one-dimensional elastic body. We first examined the uniform medium on the assumption that an impulsive excitation is given at a certain point and that periodical boundary conditions are applied. It was confirmed that no grid dispersion is observed up to the Nyquist wavenumber as the theory suggested. Non-zero values of the FWR exit only on the two diagonals in the rectangular parallelepiped spanned by w-, k- and k'-axes where w, k and k' represent angular frequency, wavenumber of the wave field and that of external excitation, respectively. We obtain a travel time curve which shows that the two impulses start propagating leftward and rightward from the set point of excitation, respectively and that the wave propagating leftward disappears at the left end of the body and appears again from the right end at the next time step, and vice versa. We also obtained the wave field in the frequency and space domains. Such a representation is not popular and appears somewhat unique but it is the essential representation of the ACROSS data. Secondly, we examined a case with a circular ring consisting of two half rings with different physical properties excited by an impulse at a point. Just by looking at FWR on the k-k' plane at a given w, we note a presence of four dominant spikes taking place in one quadrant, indicating the presence of two media accommodating two resonance wavenumbers in mutual interaction. We recognize two straight lines in the dispersion relation, suggesting that there are two different phase velocities in the medium. The travel time curve shows that the impulses generated by reflection at the boundaries propagate repeatedly coming back and forth together with the transmitted impulse between the discontinuities as expected.

Derivation of the FWR requires arithmetic operations of $O(N^3)$ for a given w in a one-dimensional case. Being more practical in a 3-D problem, it amounts to $O(N^9)$ for each w. If we take N=10^2, it becomes $O(10^{18})$. The computation of the FWR is usually necessary at many discrete frequencies spanning wide range to compute the wave form by Fourier transform of the FWR. However, the use of SEA (Sompi Event Analysis) allows us to save significantly the number of discrete frequencies to compute the FWR for deriving the arrival time and amplitude of isolated wave elements (Hasada et al., 2001). Therefore, we would be able to complete the whole calculation in dozens of days, whereas it does not appear practical at this moment. However, the present theory of computing wave field in frequency domain would be recognized to be useful within a few years, since the progress of computer technology is so fast.