S231-012 Room: 302 Time: May 24 12:00-12:15

Transformation of wave equation to its discrete equivalent in an inverse form for wave field computation

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Wave equation describes the physics law at a point (local property by differential equation) in a formalism of linear dynamic system with wave field w(t,x) as an input and excitation e(t,x) (inhomogeneous term) as an output,

e(t,x') = *delta*(x',x) D(dt,dx,*rho*(x),c(x)) w(t,x) - - - - - - - - - - (1)

where D is an operator including dt - time differentiation, dx - spatial differentiation and two independent material parameters; *rho*(x) is density (magnetic permittivity / dielectric constant), c(x) is elastic constant (inverse dielectric constant / inverse magnetic permittivity) for elastic waves (for electromagnetic wave) and *delta*(x',x) is Kronecker's delta. We write (1) by

e = Dw (local representation for all t and x) - - - - - - - - - - - - - - (1a)

By representing w(t,x) and e(t,x') by finite discrete Fourier transform, w(f,k) and e(f,k') in terms of frequency f and wavenumber k, we can treat the differentiation at the discontinuity well by means of hyper-function, and the differential equation operator D itself is converted to a matrix operator P(f,k',k) with such elements allowing arithmetic calculus. Then (1a) is converted to e(f,k') = sum_k P(f,k',k) w(f,k), which is written shortly by

e = Pw (global representation for arbitrary f) - - - - - - - - - - - - - (1b)

This is a global representation of discrete equivalent of (1a), in contrast to the local description of (1a) by means of differential equation. Since all the elements of P allow arithmetic calculus, an inverse of P(f,k',k), denoted by R(f,k,k') is existent and computed by arithmetic calculus. In other words, wave equation (1a) can be converted to w(f,k) = $\mathrm{sum}_{k'}$ R(f,k,k') e(f,k') via (1b), and it is written by

w = Re (global representation for arbitrary f) - - - - - - - - - - - - - (2)

This global representation is a description of a linear dynamic system with excitation e(f,k') as an input and wave field w(f,k) as an output. Equation (2) is an inverse representation of wave equation and it may be called 'inverse wave equation'. The R is such a quantity that describes 'frequency wavenumber response characteristics' of the wave-accommodating medium as a linear dynamic system, and it is denoted by FWR hereafter. When FWR is converted to space description, R(f,x,x'), by Fourier transform, we obtain transfer function between the source and receiver, which is nothing but the ACROSS observables. It is identified with Green function, if it is transformed to time domain, R(t,x,x') by means of Fourier transform or by SEA (Sompi event analysis, Hasada et al 2000).