

A stability analysis of a conduit flow model for lava dome eruptions

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Periodic variations in magma discharge rate and ground deformation have been commonly observed during lava dome eruptions (e.g., the 1922-2002 eruption of Santiaguito, the 1995-1997 eruption of Soufriere Hills and the 1991-1995 eruption of Unzen). Such periodic behavior has been explained by coupling effects of conduit flow with variable magma viscosity and pressure in magma chamber with elastic wall. It has been suggested that the magma flow rate and the chamber pressure tend to oscillate when effective viscosity of magma decreases as magma flow rate increases.

We performed a stability analysis of a conduit flow model proposed by Barmin et al., [2002]. The model incorporates coupling effects conduit flow and chamber pressure as well as the temporal and spatial changes of the effective viscosity controlled by the kinetics of crystallization, and it has successfully reproduced the periodic variations in magma flow rate and chamber pressure. We found that the model is reduced to a dynamical system in which the time derivatives of the magma flow rate (dQ/dt) and the chamber pressure (dP/dt) are functions of Q and P evaluated at some earlier time $t-t_d$. Here, the time delay t_d represents the time for the viscosity of fluid particle to increase in a conduit. We also found that the dynamical system with time delay is approximated by a simple two-dimensional dynamical system of Q and P where t_d is given as a parameter.

The results of our linear stability analyses for these dynamical systems indicate that the transition from steady to periodic flow depends on nonlinearities in the steady state relation between Q and P . In Barmin's model, the steady state relation shows a sigmoidal curve in Q - P phase plane; its slope has negative values at intermediate flow rates. The steady state solutions become unstable, and hence P and Q oscillate periodically, when the negative slope of the steady state relation ($[dP/dQ]_S$) exceeds a critical value; that is $[dP/dQ]_S$ is less than $-t_d G/2V_{ch}$, where V_{ch} is the chamber volume and G is the rigidity of the chamber wall. We also determined periods of oscillation at the critical condition for the periodic variations to occur.

The nature of the periodic behavior of lava dome eruption is roughly controlled by LV_{ch}/r^4 (referred to as system capacity). When the system capacity is small, the periodic behavior is accounted for by a sinusoidal oscillation (Type B limit cycle) and transition from the steady flow to the periodic flow is gradual (super-critical Hopf bifurcation). When the system capacity is large, the periodic behavior is characterized by a series of peaks with slow increase and rapid decrease (Type A limit cycle) and transition from the steady flow to the periodic flow is discontinuous (sub-critical Hopf bifurcation).

The present results provide reasonable interpretations for the periodic behavior of volcanic eruptions. In geophysical observations such as measurements of ground deformation and magma effusion rate, two types of periodic variations are observed. In the 1995-1999 eruptions of Soufriere Hills, periodic variation in ground tilt had a short period of a few ten hours and showed sinusoidal patterns. In contrast, in the 1992-2002 eruptions of Santiaguito, periodic variation in magma effusion rate had a long period of a few years and showed cyclic patterns of a series of peaks with slow increase and rapid decrease. The difference between these two periodic variations can be explained by the difference in the capacity of the systems which cause the concerned periodicity; the system capacity of Soufriere Hills is small, while that of Santiaguito is large.