

Diffusion-controlled melting in granitic systems II: modeling with two moving boundaries in the binary eutectic systems.

Tadao Nishiyama[1]; Takashi Yuguchi[2]; Hiroshi Isobe[3]

[1] Earth Sci., Kumamoto Univ.; [2] Grad. school of sci. and tech. Kumamoto univ.; [3] Dept. Earth Environ. Sci., Fac. Sci., Kumamoto Univ.

There are several ways to determine the diffusivity of components in the silicate melt. This paper presents a new simple method to estimate the diffusivity by employing a solution of the Stefan problem with two moving boundaries in the binary eutectic systems. In the companion paper (Yuguchi et al.) we have presented experimental results of the diffusion-controlled melting in the granitic systems. The melt layer developed between the two crystals (albite and quartz in the Ab-Qtz-H₂O system, and orthoclase and quartz in the Or-Qtz-H₂O system) can be considered as a melt formed between the sandwich couple. It grows both sides, consuming the two minerals. Mathematically the melt growth can be described as a Stefan problem with two moving boundaries. The solution of this problem has already been given by Christensen (1977), which can be applicable to our problems with some further considerations. In the melt layer there are concentration gradients of SiO₂, Al₂O₃, and Na₂O in the Ab-Qtz-H₂O system and those of SiO₂, Al₂O₃, and K₂O in the Or-Qtz-H₂O system, respectively. In the ternary plots, the compositions of the melt make a linear trend from albite to quartz in the Ab-Qtz-H₂O system and that from orthoclase to quartz in the Or-Qtz-H₂O system, respectively. This linearity means that diffusion in the melt layer is binary in nature in both systems, i.e., diffusive components are albite component, orthoclase component and quartz component, instead of SiO₂, Al₂O₃, Na₂O, and K₂O. Thus we model the melt growth as a Stefan problem with two moving boundaries in the binary eutectic systems.

1. Method

In the following treatment, volume effects are neglected, and then the mathematical description will be independent of the choice of the fixed point for the x (distance) axis. Here we will take the original contact of the sandwich couple as x=0. Christensen (1977) gave the solution for the composition of the melt bounded by two boundaries located at x=X_a and x=X_b, which are time dependent.

Boundary conditions are such that means solid phases (minerals) have fixed compositions of CA and CB, and diffusion does not take place beyond the two boundaries.

This result can not be directly applied to our experimental results, because the precise location of the original contact cannot be known after the run and hence X_a and X_b are unknown. To overcome this difficulty we use a linear approximation which is also given in Christensen(1977) as follows:

$$2P_a(P_a + P_b) = (C_a - C_b)/(C_A - C_a) = A$$

$$2P_b(P_a + P_b) = (C_a - C_b)/(C_b - C_B) = B$$

$$P_a = -X_a/(4Dt)^{1/2} \text{ and } P_b = X_b/(4Dt)^{1/2}$$

C_a and C_b represent composition of the melt just in contact with mineral A and B, respectively.

By these relations we can easily calculate p values from measured concentrations of the melt in the run products as follows:

$$P_a = (A^2/2(A + B))^{1/2} \text{ and } P_b = A/2P_a - P_a$$

Inserting these values into the following relation:

$$X_b - X_a = (P_a + P_b)(4Dt)^{1/2},$$

we get the value of diffusivity D, because the magnitude of (X_b-X_a) can be given as a thickness of the melt layer even though we do not know each value of X_a and X_b.

2. Results

The calculated results differ only by a factor of two or so from those defined by the parabolic growth rate law in the companion paper. Our new method is more precise than the method based on the parabolic growth rate law and has a merit to get the value of diffusivity based on only one experiment, compared to the other method requiring at least three experiments with different duration.