

Sensitivity of Coda Wave Energy

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1. Introduction

To characterize the random inhomogeneity in the lithosphere the scattering coefficient, the reciprocal of mean free distance, has been widely used by studying coda wave envelope. Since scattering coefficient stochastically relates to the number distribution of discrete scatterers or solid-angle average of energy flux of the scattered waves, we can use this value as a measure of random inhomogeneity in the earth. Assuming the scattering coefficient as a spatially varying value, tomographic inversion of scattering coefficient have been done to relate the coda envelope amplitude (or amplitude pattern) with the spatial change of the small scale inhomogeneity. However, their approaches strongly rely on the single scattering theory. This means that the difference between theoretical and observed coda wave amplitude is always mapped to the isochronal scattering shell (ISS) which defines the scatterer distribution that can contribute to the coda wave amplitude. ISS generally forms a ellipsoid in the uniform space for the case of single-mode scattering. In other words, coda waves under the single scattering approximation have a sensitivity localized on the ISS. If multiple scattering in the medium is enough small, the above assumption is right. However, the adequacy of the assumption is not well known.

In the present study, we measure the sensitivity of coda wave energy to the perturbation of scattering coefficient through the derivation of sensitivity kernel which relates the energy perturbation with changes of scattering coefficient based on the multiple scattering theory.

2. Derivation of Sensitivity Kernel of Coda Wave Energy

In this study, we assume that the isotropic multiple scattering process of scalar wavefield for characterizing coda wave energy. Multiply-scattered scalar wave energy from the seismic source can be represented by the integral form of the radiative transfer equation (RTE). We consider the spatial weak perturbation of the scattering coefficient from the uniform media. By solving perturbation problem on the RTE, we can prove that the perturbed energy density also obeys RTE having spatially distributed source. By changing source and receiver using reciprocity relation, finally we represent the perturbation of the energy density as a weighted integration of perturbation of scattering coefficient weighted by the sensitivity kernel with respect to the space. This sensitivity kernel represents the spatial distribution of volumes where coda waves are affected by the inhomogeneity of the scattering coefficient at a specific lapse time.

3. Evaluation of the Sensitivity Kernel using Numerical Solution

We can evaluate the sensitivity kernel numerically if we know the solution of the RTE having constant scattering coefficient. Snap-shot of the sensitivity kernel at the several lapse time are calculated by using numerical solution of RTE. At the earlier lapse time, perturbation of observed energy density has very strong sensitivity on the ISS which is an ellipse whose foci are source and receiver. This well agrees the fact that the energy density at shorter lapse time before the mean free time is well approximated by the single scattering theory. Sensitivity kernel amplitude at out of the ISS is always zero because of the causality. As lapse time increase, the sensitivity kernel amplitude on the ISS relatively decreases compared to the inside of the ISS. Especially, sensitivity at around the source and the receiver has very high amplitude. This high amplitude implies that the later coda envelope is mainly affected by the inhomogeneity at around source and receiver.