Numerical Simulation of the Kinetics of Crystallization in a Binary Eutectic System under One-dimensional Conductive Cooling

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Igneous phenomena consist of several elemental processes such as advection, thermal and mass diffusions, and phase transition (i.e. crystallization and vesiculation of volatile components). Non-linear couplings of these processes cause various trajectories of temperature change, and it should be recorded as igneous textures such as crystal number density. This means it may be possible to extract the quantitative information of the elemental processes in igneous phenomena, using the igneous textures. To do that, it is necessary to quantitatively relate the textures with the elemental processes through mathematical models. As an attempt to this goal, we developed a mathematical model of the kinetics of crystallization in a binary eutectic system under one-dimensional conductive cooling and show its numerical solutions for comprehensive ranges of controlling parameters.

Our mathematical model takes into account thermal and mass diffusions and the kinetics of crystallization. Diffusion and crystallization are coupled through the source terms of the equations of diffusion. The kinetics of crystallization is described by the model of Toramaru (2001). This model has consistency with experimental results and field observations when crystallization starts from the initial temperature above liquidus. Numerical analysis was conducted, using the finite volume method and the finite difference method for the equations of diffusion and the kinetics of crystallization, respectively. The calculation algorithm is the combination of explicit and implicit methods. First, the kinetics of crystallization is explicitly calculated. The result of this step yields the source terms of the equations of diffusion. The equations of diffusion are, then, solved with implicit method in order to avoid the limit of time step length. The target region of the calculation is a whole region consisting of magma and country rock. The ends of the region are enough far from the contact boundary between magma and country rock, so that both of thermal and mass diffusions effectively never reach to the ends. Uniform temperatures are initially given to magma and country rock. The calculation lasted until a fixed region crystallize completely.

Assuming appropriate values of dimensionless controlling parameters of the kinetics of crystallization, the calculation was conducted with Stefan number (St) of 0.0 to 0.3, which represents the effect of latent heat, and Lewis number (Le) of 10^2 to 10^5, which is the ratio of thermal diffusivity to mass diffusivity. The results show the cyclic distribution of number density, mean radius, and volume fraction of crystals when Stefan number is 0.2 or more, or Lewis number is 10^2 or less. One example in the figure shows that both phases of A (liquidus phase) and B have cyclic variations in crystal textures. The cyclicity of phase A is caused by the effect of cyclic increase in super saturation after crystallization of phase B (eutectic oscillation: Toramaru (2006, JPGU meeting)). By means of exhaustive parametric study, we have found the nature of this eutectic oscillation such as 1) required condition for appearance of the eutectic oscillation, 2) geometric nature of the cyclic distribution and its dependence on the controlling parameters, 3) appearance of self-similarity in the cyclic distribution.

We can find cyclic change in igneous textures in several fields such as dikes in Hirado, Nagasaki and a sill in Sado, Nigata. Our results will provide important insights for quantitative understanding of mechanism in formation of these cyclic textures.

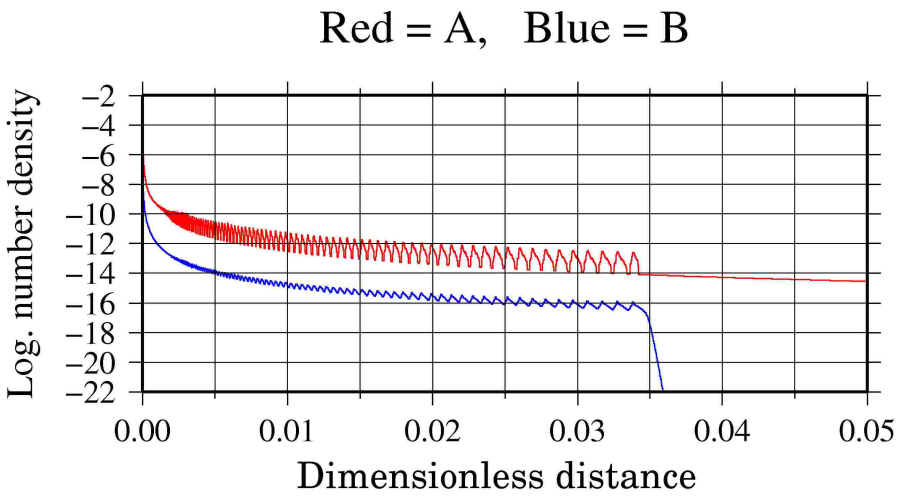

Fig. Cyclic distribution of crystal number density. St=0.2, Le=10^4.