

The Raupian manifold

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A theoretical morphospace is a morphological set of organic forms that are possibly generated by a theoretical morphologic model. It is usually depicted as a geometric hyperspace in which a particular shape is plotted with orthogonal axes chosen to represent morphological traits. The amount of the space occupied by a particular taxonomic group provides an assessment of disparity of the group. Among organisms being studied using theoretical morphospace, molluscan shell form is the most popular target because of its geometric regularity. In particular, Raup's (1966) model has been widely applied to evolutionary morphology of mollusks, in which the manner of shell coiling is defined by whorl expansion rate, relative width of umbilicus, and rate of translation of the whorl along the coiling axis. The three-dimensional morphospace constructed on the basis of the Raupian model is the most famous morphospace ever illustrated and is often called 'the Cube'.

However, the Raupian Cube is not convenient to document the range of actual form with a large whorl expansion rate such as limpet or bivalve shell forms because a slight change in shape requires a great change in the whorl expansion rate in these animals. That is, the value of the whorl expansion rate exponentially increases and diverges as the shells of limpets or bivalves reduce their shell convexities. The Raupian model can be modified to make it more suitable for analyzing shell forms of limpets or bivalves, but the modified version would be in turn unsuitable for analyses of ammonoid and gastropod shell forms in which the whorl expansion rate is fairly small. This is not a particular defect of the Raupian model but a common problem among theoretical morphologic models for defining a set of shell forms that can be approximated by logarithmic helicospirals. Such a property of spiral shell models inevitably makes the Euclidian distance in the morphospace unreliable as a metric for disparity if a set of hypothetical forms are mapped in the Cartesian coordinate system such as the Raupian Cube.

Here, I introduce a three-dimensional hyperspherical theoretical morphospace for the Raupian model to overcome the problem of parameter divergence. In such a morphospace, a particular shape is represented by a single point on the closed hypersphere on which a completely flat shape with the infinite whorl expansion rate is plotted at the North Pole of the hypersphere. A neighborhood of a particular point of this morphospace can be approximately regarded as the Euclidean space since a hypersphere is a kind of manifold. I would name the hyperspherical theoretical morphospace the Raupian manifold. If one of the three parameters is fixed to a constant value, the cross section of the Raupian manifold is depicted as a two-dimensional sphere on which each shape is represented by spherical coordinates, i.e., the angles of latitude and longitude. The radius of the two-dimensional sphere constructed by changing two of the three Raupian parameters depends on the value of the remaining parameter.