

# Fractal geometry of fault populations and fault zones; their relations with earthquakes

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## I. FRCTAL GEOMETRY OF FAULT POPULATIONS

Existing faults are the fractal backbone of earthquakes. Faulting as a dissipative process should be formulated as evolution laws.

### I-1. Fractal size frequency of faults (Otsuki, 1998, GRL)

The cumulative number  $N$  of faults with displacement larger than  $D$  among total number  $N_t$  is approximated by the equation below.  $N/N_t = [1 + (D/D_c)^A]^{-B_s/A}$  — (1),

$D_c$ : the characteristic slip,  $A$ : a parameter of ductility,  $B_s$ : fractal dimension. Let  $N_d$  and  $D_d$  as number density and slip density of faults, then equations below holds.

$$A = 0.495 * (D_c/N_d)^{-0.335} \quad R^2 = 0.962 \quad \text{---(2)}$$

$$B_s = 1.695 * [D_d/(D_c N_d)]^{-0.253} \quad (R^2 = 0.980) \quad \text{---(3)}$$

$D_c N_d$  is a quantity proportional to the ratio of dissipative energy density to elastic strain energy density, resembling Reynolds number. The term  $D_d/(D_c N_d)$  is interpreted as input energy density measured by dissipative energy density, and  $B_s$  decreases as input energy density. Note pre-exponential constant to be very close to the golden section ratio.

### I-2. Spatial distribution of faults (Goto & Otsuki, 2004, GRL)

This is a multi-fractal problem of slips which are supported by the spatial distribution of faults. The information dimension of the distribution of all faults in any population is very close to 1, indicating nucleation spatially at random. The information dimensions  $B_0$  of the fault populations with  $D$  larger than  $D_c$  is approximated as,

$$B_0 = 1.12 * (D_c N_d)^{0.0397} \quad R^2 = 0.868 \quad \text{---(4)}$$

This suggests that the spatial distribution of faults which can grow spontaneously is determined only by the non-dimensional material property  $D_c N_d$ .

The information dimensions  $B_1$  of the spatial distribution of fault slips follow the equation below.

$$B_1 = 0.865 * [D_d/(D_c/(D_c N_d))]^{-0.0683} \quad R^2 = 0.904 \quad \text{---(5)}$$

$B_1$  also decreases as input energy density measured by dissipative energy density.  $B_s$  and  $B_1$  is tied by an universal law as,

$$B_1 = 0.763 * B_s^{0.219} \quad R^2 = 0.950 \quad \text{--- (6)}$$

## II. FRACTAL GEOMETRY OF FAULT ZONES

### II-1. Hierarchical self-similar fault zone geometry (Otsuki & Dilov, 2005, JGR)

The inspection into growing experimental faults revealed that any fault zone is composed of fault segments and jogs, and that any large segment is nested by smaller segments and jogs. Let  $L_s(i)$ ,  $L_j(i)$  and  $W_j(i)$  as segment length, length and width of jog of hierarchical rank  $i$ , respectively. They are well approximated by,

$$L_s(i) = 0.343 * L_s(i-1)^{0.999} \quad R^2 = 0.956 \quad \text{---(7)}$$

$$L_j(i) = 0.0935 * L_s(i-1)^{1.00} \quad R^2 = 0.861 \quad \text{---(8)}$$

$$W_j(i) = 0.0456 * L_s(i-1)^{0.642} \quad R^2 = 0.691 \quad \text{---(9)}$$

Similar equations hold for 20 surface seismic faults in the world, indicating them as universal laws covering very wide size range, orders from mm to 100km.

### II-2 Derivation of two empirical laws in seismology

Gutenberg-Richter law and another empirical law that  $M_0$  of main shocks is proportional to the seismic nucleation size to the power of 3 can be derived from the hierarchical self-similar fault zones with the plausible assumptions.

### II-3 Derivation of surface fracture energy (Otsuki, 2007, GRL)

Comminution is most intensive in jog areas, and the fractal dimension  $D$  of microcrack distribution is 1.56 (probably a universal constant). Incorporating this and the fault zone geometry, the size dependence of surface fracture energy  $G_s$  can be derived as

$$G_s = C_1 * (L_0/x_0)^{(D-1)} \quad C_1: \text{const.} \quad \text{---(10)}$$

indicating that  $G_s$  is determined by the length of fault  $L_0$  scaled by the characteristic smallest grain size of fault gouge.  $G_s$  of jogs of the 1st, 2nd, 3rd rank are about 2, 1, 1/2 times of the mean  $G_s$ .

### II-4. Size dependence of static stress drop (Nakamura & Otsuki, Poster)

Nadeau & Johnson (1998, BSSA) found an empirical relation for the size dependence of stress drop. The relation between segment length  $L$  and slip  $D$  of surface seismic faults is consistent with their relation as

$$D/L = 0.0253 * L^{-0.665} \quad R^2 = 0.964 \quad \text{---(11)}$$

depicting the larger fault, the smaller stress drop. This will be explained by fractal distribution of slip density on a fault plane.