Geophysical Data Inversion: From the Least Squares Method to a Bayesian Approach

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In geophysics, model estimation is done on observed data. Unlike experimental data, observed data are always insufficient, inaccurate and inconsistent. How can we estimate the optimum model from such data? What is the criterion for optimality? This is the main subject of inverse problems in geophysics.

Now we consider a linear observation equation with n data and m model parameters. If the rank p of the coefficient matrix is equal to n and m, we have only one exact solution. In other cases, we need some criterion to define a particular solution. For example, if n is greater than m=p, we obtain the least squares solution under the criterion that the square norm of the residual vector is minimum. If m is greater than n=p, we obtain the minimum norm solution under the criterion that the square norm of the solution vector is minimum. The solution by Lanczos' inverse matrix, constructed out of the p positive eigenvalues and the corresponding eigenvectors of the coefficient matrix, gives a general solution that includes the above classical solutions as special cases. This solution satisfies the criterion that both norms of the residual vector and the solution vector are minimum.

In actual problems, the solution by Lanczos' inverse matrix does not always give good results, because observed data are contaminated by noise. For practical use, Jackson (1972) and Wiggins (1972) modified this solution, and proposed a sharp-cutoff approach by imposing the constraint that the variance of estimation errors must be less than a maximum allowable variance. One of the problems in this approach is how to set the maximum allowable variance. On the other hand, regarding prior information about model parameters as observed data, Jackson (1979) derived the minimum variance solution, which minimizes the sum of the variance of estimation errors due to data noise and the variance of estimation errors due to poor resolution. From the viewpoint of probability theory, this solution is nothing but the maximum likelihood solution, which maximizes the posterior probability density obtained by incorporating prior information into observed data with Bayes' rule (Jackson & Matsu'ura, 1985). The problem of this solution is how to properly choose the relative weight of prior information to observed data.

The introduction of the entropy maximization principle into statistical inference (Akaike, 1977) perfectly resolved the problems for the proper choice of the maximum allowable variance in the sharp cutoff approach and the relative weight of prior information to observed data in the minimum variance solution. In practice, we can apply AIC (Akaike, 1974) to the former case (e.g., Noda & Matsu'ura, 2009) and ABIC (Akaike, 1980) to the latter case (e.g., Yabuki & Matsu'ura, 1992; Matsu'ura et al., 2007).

Anyway, through such history, the essential problem of geophysical data inversion has been theoretically solved. However, in application to actual data, the inversion theory does not always work so well. One of the reasons is in the imperfection of theoretical models used for analysis. The modeling errors are usually treated as random errors in observation equations, but actually they are systematic errors. If observation errors are large enough, the effects of modeling errors are negligible. At the present day, we can obtain highly accurate dense geophysical data by exciting progress in observation technology. Then, we cannot ignore the effects of modeling errors, which will cause some serous bias in inversion results. In order to resolve this problem, we need to use much more realistic theoretical models for inversion analysis and/or construct observation equations for the specific part of observed data to be explained by theoretical models (e.g., Hashimoto et al., 2009).