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Waveform inversion for source processes in an incomplete model (1): Importance of modeling error and derivation of its expression

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Inversion analyses are important for various fields. In particular, inversion analyses play a central role in solid earth sciences. In the inversion analysis, we extract information of observed data based on observation equations. When observed data are linearly related to quantities to estimate, observation equations are expressed as

d = Ha + e

where, d is a data vector, a is a model parameter vector to estimate, H is a coefficient matrix that relates observed data with model parameters, and e is an error vector. We can rewrite the above equation as

e = d - Ha

This equation means that the error e is defined by the sum of the observation error and the modeling error. In almost all inversion analyses, however, the term of modeling error has been neglected. Neglect of modeling error is no problem, if the observation error is large. Due to the development of observation instruments, however, the accuracy of observation is now very high. In this situation, Neglect of modeling error is problematic.

In many inversion analyses, the error e has been commonly assumed to be independent of each other. However, the modeling error usually have significant covariance terms. So, we must develop the conventional method of inversion analyses.

For error-free data, observation equation is written as

d0 = H0$*$a0

for linear cases. Here, d0 is a true data vector, a0 is a true model parameter vector, and H0 is a true coefficient matrix. In fact, however, we can never know these true values. This is why, the above equation should be rewritten as

d+d' = (H+H')(a+a')

where the prime represents the error. Neglecting the higher order term of the errors, we obtain the expression of error e:

e = H'a + Ha' - d'

By taking the average of the product of e and transpose of e, we can obtain the expression of covariance matrix of the error e.

If, H', a' and d' are independent of each other, the covariance matrix of the error e is the sum of the covariance matrix of each term:

E(e) = E(H') + E(a') + E(d')

For the second and third terms, it is easy to obtain the explicit expressions based on the law of propagation of errors. It is not easy to evaluate the third term in general, but we found a way applicable to many cases. We can also deal with errors in deriving the observation equation (e.g., discretization error) within the same framework as above.

We have succeeded in deriving the general expression of modeling errors for the linear case. We will show the importance of modeling errors for actual inversion analyses of seismic waveform data (Yagi and Fukahata, this meeting).