Crack diffusion equation driven by pore crash in the theory of fluid filled porous media.

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(1) How and what cause the low-frequency tremors and slow earthquakes? Many people are thinking that the fluid within the deep earth's crust may play an important role. In order to advance such a view, I have studied analytically an ideal case of quasi-static crack development in the Gassmann-Biot theory of porous media. Presently, we have found that it is crack diffusion phenomena that result in the slow-earthquake activities.

(2) We use the extended Gassmann-Biot theory, in which the fluid is compressible (Rice and Clearly, 1976, Review of Geophys., Nagumo, 2008). The crack will be treated by zeta, which is introduced by Biot (1956, 1962) as the divergence of the displacement of the squeezed-out water from the pore spaces. This is because the squeezed-out water creates cracks and full-fills the space until it is drained out of the whole system of the porous media (Nagumo, 2008). Therefore, the problem to be solved will be to clarify the behavior of zeta.

(3) Firstly, we will derive zeta-equation. The field of the fluid stress (inverse sign of the pore-water pressure) is described by the basic coupled equations for the two unknown variables of fluid stress and zeta(Nagumo, 2008). The one is the equilibrium equation of the total media; the other is for the Darcy flow equation. From these basic coupled equations we derive a zeta-equation, which is inhomogeneous equation, where the inhomogeneous term is given by the fluid stress change.

(4) Mathematically, the form of this equation is the same as the heat conduction equation with generating source (Carslaw and Jaeger, 1948, Sneddon, 1951). This property is important to understand the crack development phenomena. Physically, the crack development is a phenomenon of diffusion driven by the pore-water pressure change.

(5) Then, using the relation of the porosity change and the fluid-stress change (Nagumo, 2008), we have another zeta-equation, in which the inhomogeneous term is the porosity change.

(6) Here, in the case of earthquake faulting, the porosity change will represent pore crash in the faulting. Thus, this equation reveals us such a process that, when cracks crash in the earthquake faulting, the pore-water pressure increases and the crack develops. In short, it may be referred to as the crack diffusion driven by pore crash.

(7) In order to solve the basic coupled equations, an equation for the single variable zeta is obtained by eliminating the fluid stress terms. This equation is an inhomogeneous bi-harmonic equation of the single variable zeta. Mathematically, the form of the equation is similar to that of the Airy stress function in the two dimensional elastic stress system (Sneddon, 1951, Fourie Transform, Section 43.4, Equ. (8)). Therefore, it can be solved.

(8) Among many possible solutions, a solution which satisfies the homogeneous equation is instructive for understanding the process of crack development. Mathematically, zeta satisfies the homogeneous equation in which the source is included in the diffusion term. Physically, it indicates the feedback effect of the pore-water pressure change to the crack development through the whole system of the porous media.

(9) The homogeneous zeta equation will be solved by using Laplace transform with respect to the time, and Fourier transform with respect to the space (see Sneddon, 1956, p.169), where the initial as well as the boundary conditions are automatically introduced in the transformation.

(10) The migration speed of the periodic solution is dispersive (see Carslaw and Jaegr, Conduction of heat, 1948, p. 48). Thus, we see such nature that the lower the frequency is, the slower is the speed, and the higher the frequency is, the larger is the attenuation. Further studies will be continued towards the dynamic crack propagation equation. Ref.Nagumo(2008)Proceedings of the 118th SEGJ Conference, pp. 5-8, 267-270.