

## Probabilistic bases of the frequency-magnitude scaling shown by laws such as Gutenberg-Richter formula

# Eiji Tokunaga[1]; Tetsuo Takanami[2]

[1] Economics, Chuo Univ.; [2] ISV, Hokkaido Univ

A branching aftershock sequence (BASS) model was published as an alternative to the widely studied epidemic aftershock type of aftershock sequence (ETAS) model (Turcotte et al., 2007). The BASS model is the self-similar limit of the ETAS model (Abaimov et al., 2009) and explains the concepts of parent earthquakes and daughter earthquakes as well as the Gutenberg-Richter's scaling for aftershock sequences following the idea of self-similar branching which was developed in studies of drainage networks (Tokunaga, 1978). This model utilizes a modified form of Bath's law in addition to Omori's law. Our study tries to corroborate the self-similarity of seismic activities by setting up a system which approximates the most probable state of magnitude-energy relations in the 4D space. Accordingly, we prescribe here the system by the relation between the number of aftershocks and the energy of the earthquake wave, focusing the fact that moment magnitude  $M_w$  of earthquakes is usually represented in terms of two digits rounded to one decimal place. In the approach, we divide a time-space into  $n \times 4D$  cells and assume that in one cell, either one or no earthquake occurs. And fix the proportion of  $n$  to the number of cells in which earthquakes occur. We also assume that energy as a whole approximates asymptotically to a fixed value. Here  $n(i)$  indicates the number of aftershocks of the event with  $M_w$  of which value corresponds to  $i$  (two-digit) in a sequence of aftershocks. Extract one aftershock of the highest value of magnitude (before rounding) and one aftershock of the lowest value from the group of aftershocks of magnitude  $i$ , and add one aftershock which has the highest value in the group of magnitude  $(i-1)$  to that group and also add one aftershock which has the lowest magnitude in the group of magnitude  $(i+1)$  to that group instead. Then the total energy of the earthquake wave, which corresponds to these three values, slightly increases, while the sum of numbers of aftershocks remains unchanged. For example, such procedure for  $i = 2.0$  produces the increase of total energy slightly more than 1% of energy of one aftershock of magnitude 2.00, when aftershocks occur sufficiently covering the numbers of two decimal places in these three groups. The number of aftershocks in a group should increase as the  $i$  value decreases. Let denote the sum of wave energies of the aftershocks in the group by  $e(i)$ . Then we can obtain, after methods used in statistical thermodynamics, the most probable state of patterns of 4D cell distributions under the restrictive conditions that  $\{n(i-1) + n(i) + n(i+1)\}$  and  $\{e(i-1) + e(i) + e(i+1)\}$  are respectively constants. This state is sustained when  $n(i)/n(i-1) = n(i+1)/n(i)$ . The aftershock sequence approximates asymptotically to the state as the magnitude decreases. Those sequences of various levels form a self-similar tree in the BASS model. The amount of wave energy was calculated from  $M_w$  value by using the relation proposed by Kanamori (1977).